Exploring Mathematical Connections Between Abstract Algebra and Secondary Mathematics from the Perspectives of Mathematics Faculty and Practicing Teachers

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Introduction/Methods

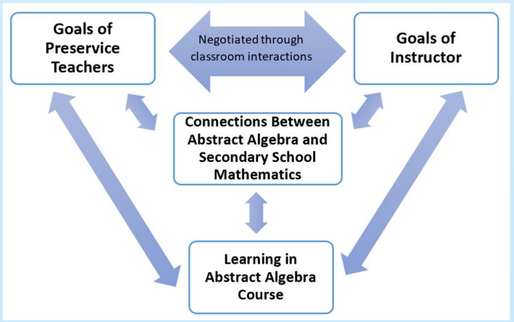
- Abstract algebra is important for preservice teachers (Wasserman et al., 2017)
- Many teachers see no relation between abstract algebra and secondary school mathematics (Christy & Sparks, 2015; Ticknor, 2012)
- Part of two-stage, qualitative dissertation research study
- Two Qualtrics surveys administered spring and summer of 2019 to gather background information for stage two:
  - Mathematics faculty (sent to 75 mathematics department chairs across the county)
  - Practicing secondary mathematics teachers enrolled in or recently graduated from graduate programs designed to strengthen the mathematical background of practicing teachers (graduate abstract algebra course part of degree requirements)

Research Questions:

1. What mathematical connections between abstract algebra and secondary school mathematics do mathematics faculty identify as important for their students to make and how do they incorporate these connections into their instruction?

2. What instructional tasks or course activities help preservice teachers establish mathematical connections between abstract algebra and secondary school mathematics while they are enrolled in an introductory abstract algebra course?

Conceptual Framework:



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Results

Results from the surveys were used to inform both interview protocols and instructional tasks used in the second stage of the research.

Top 5 Mathematical Connections from Mathematics Faculty Survey Results

Abstract Algebra Concept	Secondary School Mathematics Concept	Points
Algebraic structures (group, ring, integral domain, field) & their properties	Function and domain; identity; inverse; number systems and known operators; solving linear equations	88
Polynomial ring	Operations with polynomials and polynomial long division; polynomial vocabulary (degree, coefficients, roots, etc.); power series	41
Equivalence relation	Congruence; inequality; similarity; symmetry	39
Groups and specific types of groups	Function composition; geometric transformations and symmetries	34
Binary operator	Arithmetic operators and number systems; domain function; function composition; function transformations	31

Participants were shown and asked to re-rank the connections listed in Appendix D in *Abstract Algebra and Secondary School Mathematics: Identifying and Classifying Mathematical Connections* by A. L. Suominen, 2015, p. 99-100.

Sample Instructional Tasks:

Introductory Abstract Algebra Warm Up Activity 1

Suppose you are a 7th grade math teacher who is reviewing for the final exam with your students and you give the students time to solve the equation $\frac{x}{-8} = -7$ after you've written it on the board. How would you respond to the following? (please fill in the blank with your response and feel free to comment on your response at the bottom of the page)

You (the teacher): Okay so to solve the equation written on the board what would we do?

Student 1: Multiply both sides by -8 so the answer is x equals 56.

Student 2: So I remember us learning that a negative times a negative is a positive, but why is that the case again? I mean even just like negative one times negative one, why isn't that negative one again?

You (the teacher):

Introductory Abstract Algebra Warm Up Activity 4

You are helping your parent, who is a high school mathematics teacher, check their students' homework during your spring break while you're in college. One student writes the following:

$$(x+2)(x-3) = 6$$
$$x+2 = 6 \text{ or } x-3 = 6$$
$$x = 4 \text{ or } x = 9$$

Is the student correct? Why or why not? If they are not correct, provide a short response that you might write to the student to help them see their mistake and/or when their method would be correct.

Results Continued

How Mathematics Faculty Report Incorporating Connections to Secondary School Mathematics

How the participants currently incorporate connections between abstract algebra and secondary school mathematics	Number of participants who selected each method of incorporating connections
Verbally describe the connection(s)	37
Verbally give an example of the connection(s) in the context of secondary mathematics	30
Implement short (in-class) instructional activities that make explicit connections to secondary mathematics	19
Implement longer (outside of class) projects that make explicit connections to secondary mathematics	10
Other (most indicated N/A because they do not make connections)	10

Discussion

- During stage two, warm up activities were given both before and after the relevant abstract algebra material was taught as an adapted version of building up from/stepping down to teaching practice (Wasserman et al., 2017)
- Prospective teacher participants from the second stage of the research stated:
  - *Without the warm up activities and group interview (where participants discussed the warm up activities) they wouldn't have seen connections between abstract algebra and mathematics education*
  - *The warm up activities and group interview helped them understand why abstract algebra is required of future teachers and the warm up activities specifically show "that there is some connection to what [they were doing in abstract algebra] to what teachers do in schools"*
  - *The warm up activities were very realistic situations that will be helpful "for teachers who are going to be dealing with [those type of questions] everyday"*

References

- Christy, D., & Sparks, R. (2015). Abstract algebra to secondary school algebra: Building bridges. *Journal of Mathematics Education at Teachers College, 6*(2), 37 – 42.
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- Ticknor, C. S. (2012). Situated learning in an abstract algebra classroom. *Educational Studies in Mathematics*, *81*(3), 307 – 323.
- Wasserman, N.H., Fukawa-Connelly, T., Villanueva, M., Mejia-Ramos, J.P., & Weber, K. (2017). Making real analysis relevant to secondary teachers: Building up from and stepping down to practice. *PRIMUS*, *27*(6), 559 – 578.