Estimating Errors from Extrapolating Finite-Radii Gravitational Wave Merger Simulations to Infinite Radii

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What are Gravitational Waves?

When objects move in the fabric of space-time, their **displacement** propagates through the universe as energy in the form of **waves**. These are **gravitational waves** (GW), carrying the energy given off by **gravitational radiation** at the speed of light *c* [1]. They are notoriously difficult to detect, requiring large, incredibly sensitive instruments called laser interferometers (pictured top right) to detect the subtle variation in the space-time fabric.

Gravitational Wave Simulations

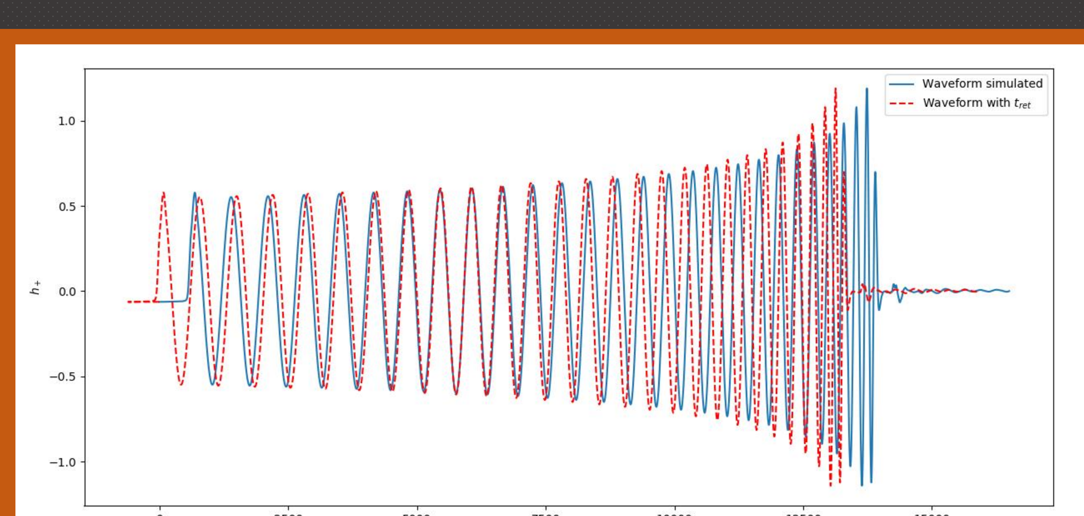
- We use the Spectral Einstein Code (SpEC) to simulate the mergers of massive astronomical object (black holes, neutron stars, etc).
- Computational simulations of GW are **crucial for identifying and cataloging waveforms** in the data collected by modern observational relativistic astrophysics [2].
- Simulations give us a repository of astronomical events to search incoming data and verify results.

The Problem

Due to **computational constraints**, gravitational wave simulations from **massive compact object mergers** can only be computed out to a small finite radius (**~60,000km-135,000km from event**). Thus, **we must extrapolate from the computed data** to see what a wave would look like at our observation at $r = \infty$. **In order to know the viability of this data for scientific use, we must determine the extrapolation method that provides the best fit with non-significant error.**

The Study

Using an extrapolation method from *Extrapolating Gravitational-Wave Data from Numerical Simulations* [3], we can apply a corrected time parameter to the simulated GW waveform, then apply **polynomial fits of different orders** to the interpolated function derived from the waveform to ultimately find the error on each extrapolation method.



Data from the simulation is in the format of **non-time corrected waveforms** in the quadrupole expansion h_+ **and** h_X. The plot to the left shows the time corrected waveform (red) over the raw data from the simulation output (blue).

Extrapolation Method

GW have two amplitudes h_+ and h_X, that can be combined into a complex signal $h = h_+ + ih_X$. We use the phase ϕ and amplitude A for analysis, defined below:

$$\phi = \arctan(h_+/h_x) + 2\pi k \qquad A = \sqrt{h_x^2 + h_+^2}$$

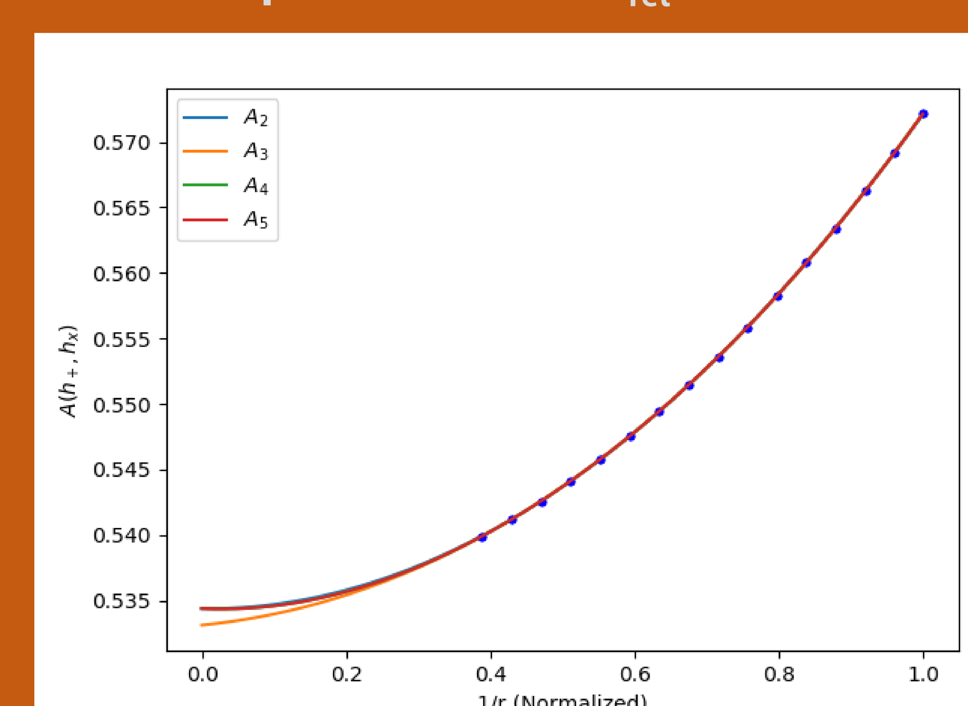
We then took the time array of the simulated waveform and applied a corrected time parameter t_c to find our relativistically adjusted retarded time, t_r, which is used to correct the waveform. This is required to account for wave propagation time.

$$t_r = t_c - r_*$$

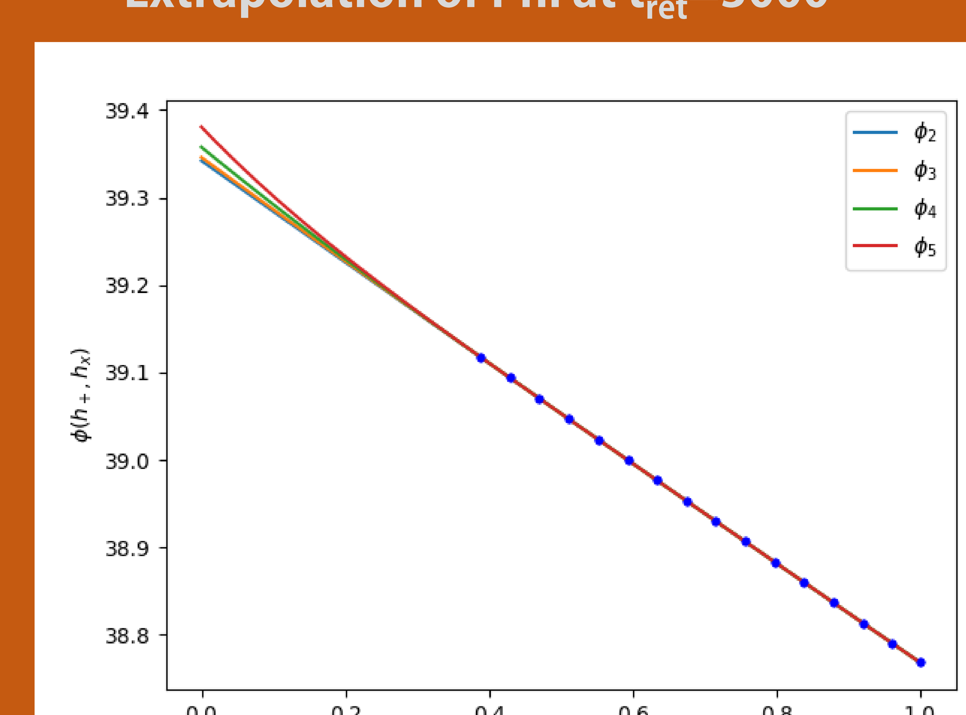
$$t_c = \int_0^T \sqrt{\frac{-1/g^{TT}}{1 - 2M_{ADM}/R_{areal}}}\,dT'$$

We then build an interpolated function with the time corrected A and ϕ data. From here, we ran polynomial fits of different orders to extrapolate the data to our observation point at $r = \infty$ ($\frac{1}{r} = 0$). We used 2nd to 5th order functional expansions around $\frac{1}{r}$ in A and ϕ to build our extrapolation.

Extrapolation of A at t_{ret}=3000



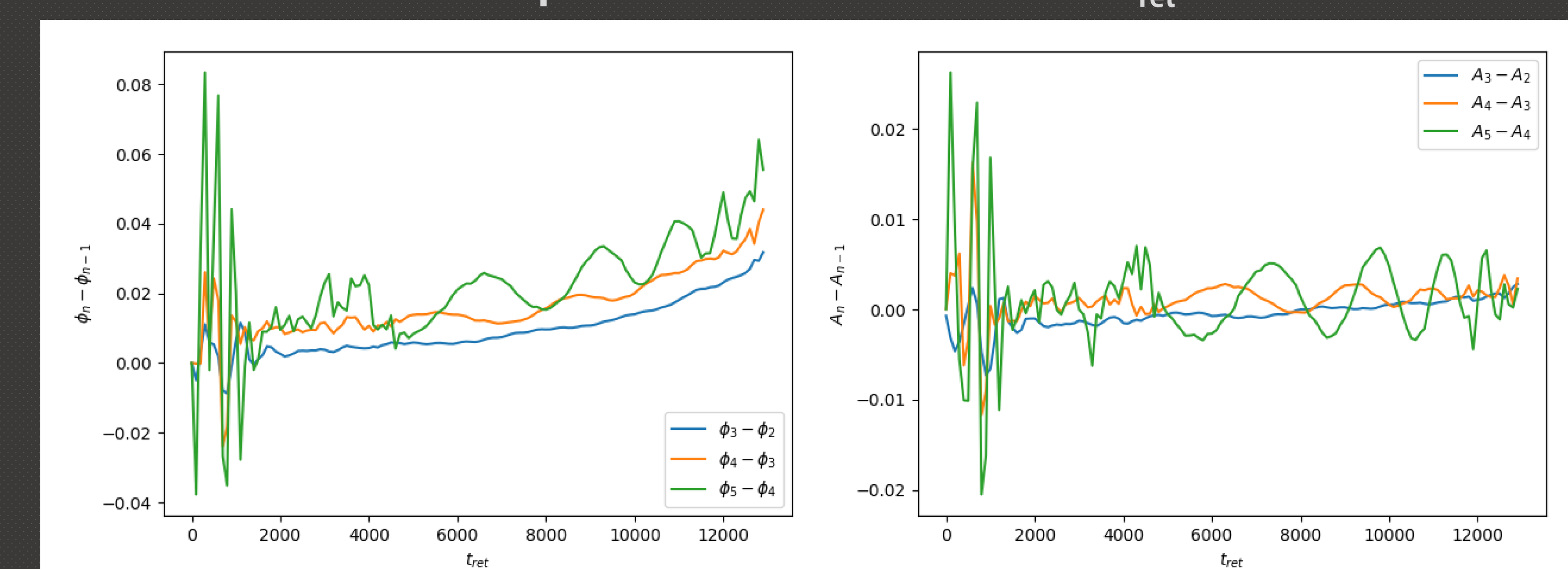
Extrapolation of Phi at t_{ret}=3000



Error Analysis

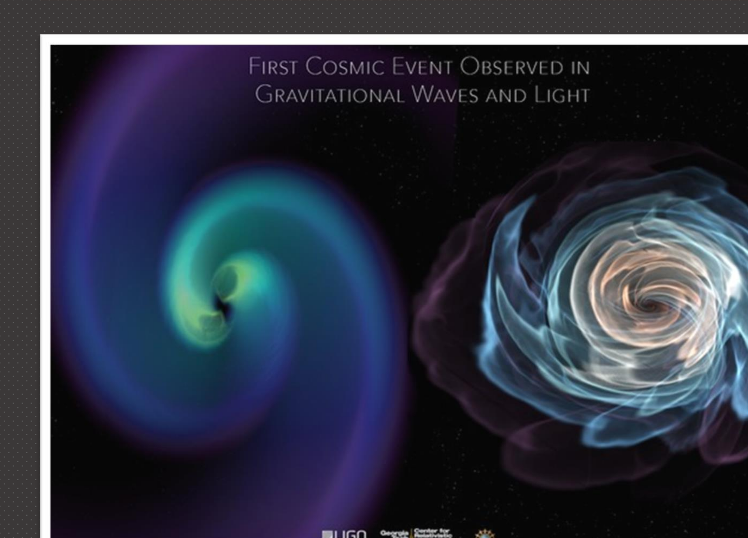
- At the highest radii, the error of the waveform is **$\sim 7 * 10^{-1}$ radians**, while the extrapolated waveform has error within **$\sim 5 * 10^{-2}$ radians**—a significant improvement.
- We analyze error on extrapolation methods by **comparing the difference between subsequent orders of functional fitting** ($A_N - A_{N-1}$ and $\phi_N - \phi_{N-1}$).
- Convergence in our A and ϕ fitting error illuminates which fitting order has the most ideal error-to-fit ratio. In the higher order fits, we can **"over-extrapolate"** where the **noise increases significantly** as the radius increases.

Error of Extrapolation Order of A and Phi as t_{ret} increases



Important Learnings

- Formatting data into computationally friendly formats (changing h_+ and h_X into the non-oscillatory A and ϕ) **increases the effectiveness of extrapolation methods**.
- **Error propagation** is the key factor to control in building methods for scientific work. In this case, we were seeking to find a **balance** between **good extrapolation and low error**.
- The usefulness of our novel choice of t_c versus applying the naïve $t_r = t - r_*$ is underrepresented in the extrapolated data. The part of the GW analyzed here are all **pre-merger**, where the choice of t_c does not make a significant difference but is important for maintaining posterity with longer timeframe studies.



References

- [1] Carroll, S. (2019). Spacetime and Geometry: An Introduction to General Relativity. Cambridge University Press.
- [2] Abbott, B… (2019). GWTC-1: A Gravitational-Wave Transient Catalog of Compact Binary Mergers Observed by LIGO and Virgo during the First and Second Observing RunsPhys. Rev. X, 9, 031040.
- [3] Boyle, M., & Mroué, A. (2009). Extrapolating gravitational-wave data from numerical simulationsPhysical Review D, 80(12).
- [4] Tim Dietrich, Sebastiano Bernuzzi, Bernd Brügmann, & Wolfgang Tichy (2018). High-Resolution Numerical Relativity Simulations of Spinning Binary Neutron Star Mergers2018 26th Euromicro International Conference on Parallel, Distributed and Network-based Processing (PDP), 682-689.