Large Language Models and the Construction of New Calabi-Yau Manifolds

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Introduction

String Theory is a theory that has been put forward to explain the four fundamental forces, Strong Nuclear Force, Weak Nuclear Force, Electromagnetic Force, and Gravitational Force. According to String Theory, there are 10 existing dimensions. Four of these dimensions are the three spatial dimensions, and the fourth is the time dimension. The six extra dimensions are curled up in complex shapes defined as Calabi-Yau manifolds.

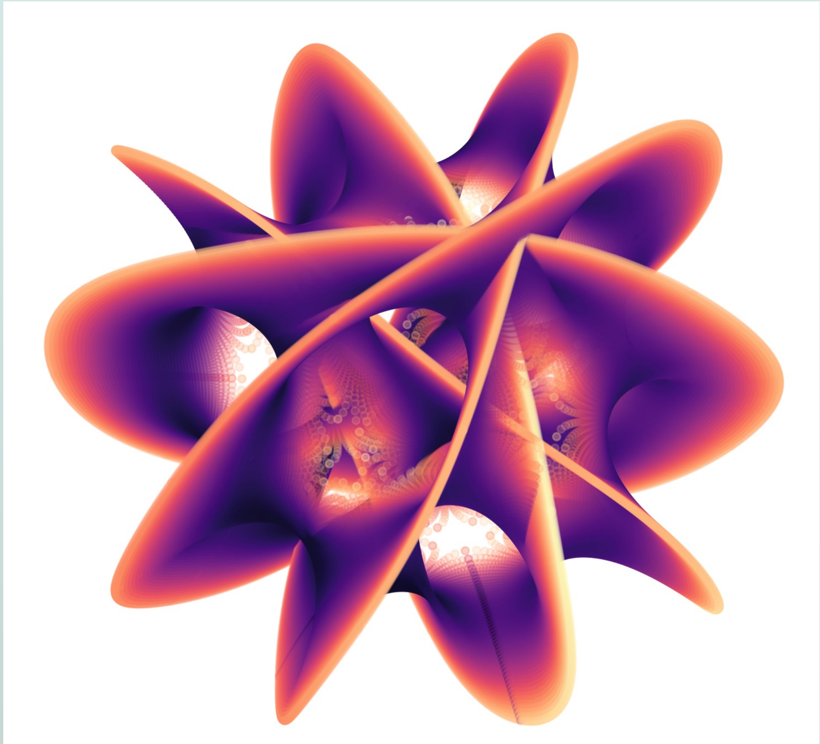

Figure 1: Visualization of the Fermat Quintic Threefold.

- Manifolds can be produced from triangulation of 4- dimensional polytopes [1,2].
- Polytopes must be reflexive, meaning that vertices can swap places with edges.
- There are over 400 million 4-dimensional reflexive polytopes that can be triangulated.
- After these polytopes are triangulated, they can be used as training sets for Large Language Models (LLMs) to analyze the resulting Calabi-Yau manifolds.
- This poster is focusing on the triangulation process

Acknowledgements

I would like to thank Professor Berglund for providing the resources needed for this project, and for providing help for when I had questions. My thanks also to Steven May and the collaboration for access to the code.

Contacts

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Methods

Project based off a collaboration of members from Meta, Axiom Math, Wisconsin and the University of New Hampshire.

- Worked with Python code that utilized CYTools, a Python package that triangulates 4-dimensional polytopes [1].
- Changed original code to triangulate specific 4D polytope input by user.
- Two triangulations that will be found and classified.
- FRST triangulations: all the vertices are connected, regular, and star triangulations are required [1,2].
- Star Triangulations: connect all vertices to interior point.
- VEX triangulations: where the vertices of the polytope are connected to one vertex. [3,4]

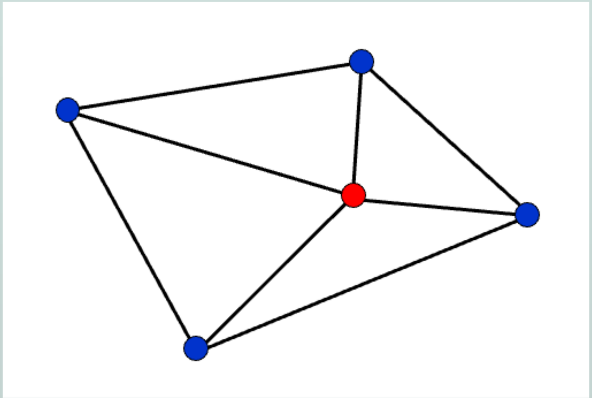

Figure 2: Star Triangulation Example

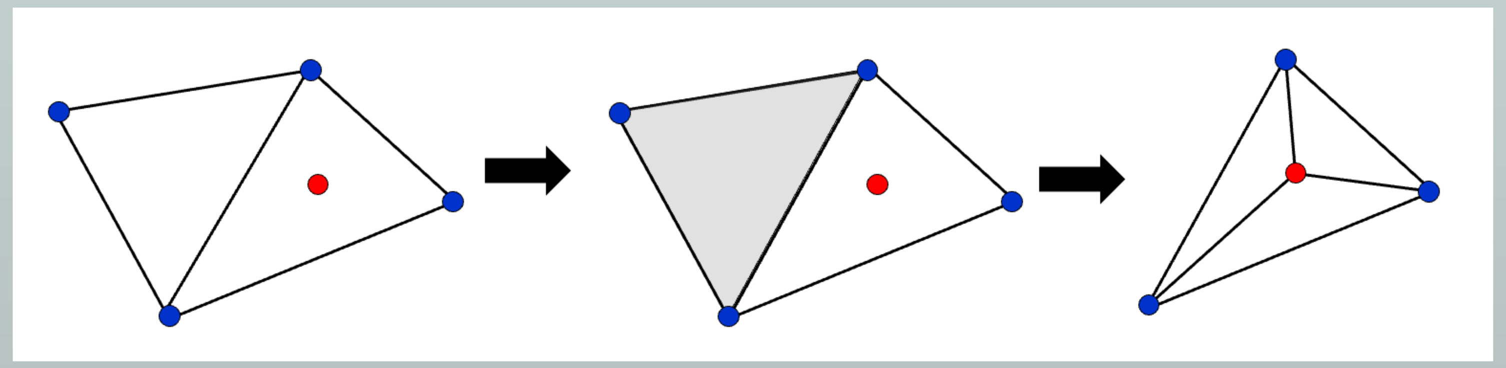

Figure 3: VEX Triangulation Example

- In a 2D polytope, the result of a VEX triangulation might result in an entirely new polytope. For a 4D polytope, it will simply result in a different triangulation.
- Code also includes different triangulation modes: "all", "fast", and "fair".
- "all": completes every possible triangulation for a specific polytope and is a long process. Once polytopes get large, it will no longer be effective due to memory issues. [5]
- "fast": does not do all possible triangulations, but more efficient.
- "fair": Compromise between "all" and "fast". Has fairer sampling than "fast" but is more efficient than "all".

Results

Code was used to analyze multiple different polytopes of different vertices and h^{11} values.

- Below is a table of triangulations of a specific polytope [5].

$$\Delta_2 = \mathrm{Conv}\begin{pmatrix} 0 & 4 & -2 & 0 & 0 & -2 & 1 \\ 0 & -2 & 2 & 0 & 1 & -2 & 0 \\ 1 & -1 & -1 & 0 & 0 & 2 & 0 \\ 0 & -2 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$h^{11} = 15$$

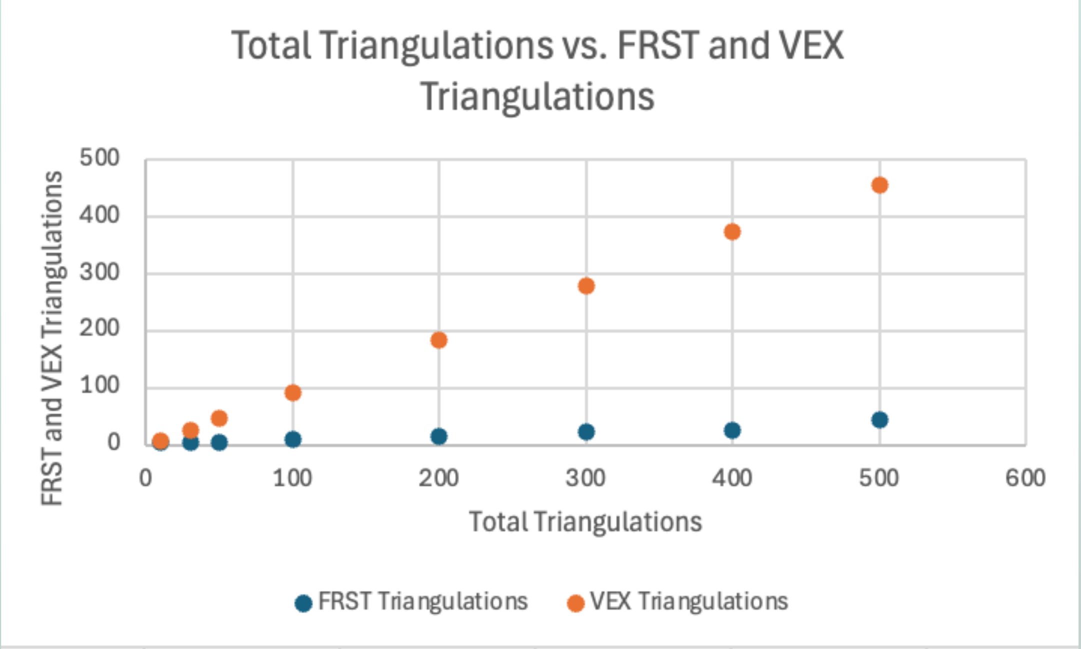

Figure 4: Number of FRST and VEX vs. Total Triangulations

- For this polytope of relatively large size, VEX and FRST triangulations increase at a linear rate as the total number of triangulations increases.
- The largest polytope may have $\mathbf{10^{979}}$ triangulations [4].
- The challenge in the future will be selecting the proper triangulation samples for further analysis.

References

[1] CYTools, https://cy.tools
[2] Jacky H. T. Yip et al. "Transforming Calabi-Yau Constructions: Generating New Calabi-Yau Manifolds with Transformers", arXiv e-print 2507.03732v2
[3] P. Berglund, T. Hübsch, "A Generalized Construction of Calabi-Yau Models and Mirror Symmetry", SciPost Phys. 4, 009 (2018)
[4] Nate MacFadden and Elijah Sheridan "Calabi-Yau Threefolds from Vex Triangulations", arXiv e-print 2512.14817v1
[5]P. Berglund et al. "Generating Triangulations and Fibrations with Reinforcement Learning", arXiv e-print 2405.21017