Computing Ricci-flat Metrics for Highly Deformed Calabi-Yau Manifolds

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Introduction

String theory seeks to explain gravity, electromagnetism, and the weak and strong nuclear forces in one theory, arguing that in addition to the three spatial and one time dimension, there are six additional dimensions, ten in total.

- String theory predicts that one-dimensional strings and higher-dimensional D-branes, explain the four fundamental forces.
- The six additional dimensions are compactified on six-dimensional Calabi-Yau manifolds so that they are very small and cannot be directly observed. Although these extra dimensions are very small, they have a strong effect on the properties of the four-dimensional universe we can observe.
- However, there are millions of possible Calabi-Yau geometries, and computing the Ricci-flat metric is necessary to determine whether a given CY geometry is realistic for our universe.
- The Ricci-flat metric is computed by numerically solving the complex Monge-Ampere partial differential equation on CY manifolds, which has no analytic solution.
- The parameter ψ controls the shape of the manifold, and at large ψ, such as 1000, the geometry becomes highly deformed, and computing the Ricci-flat metric becomes even more complex.

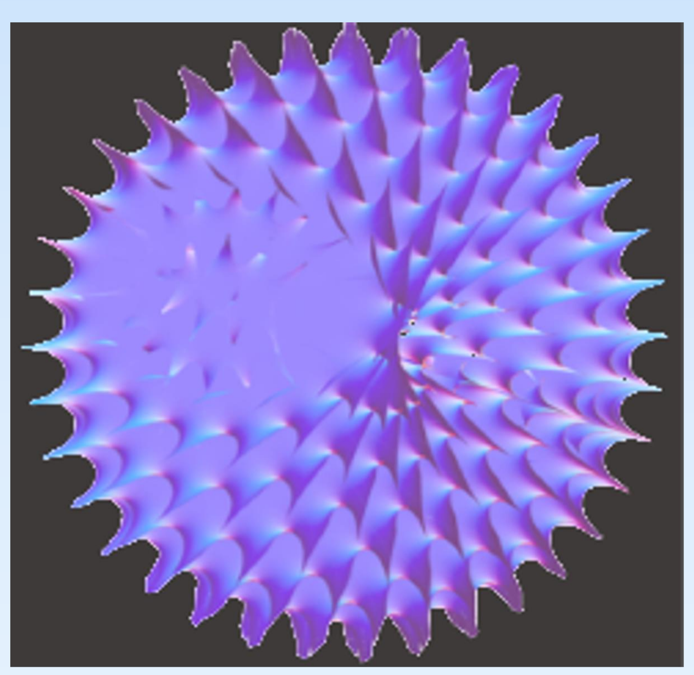

Figure 1: Projection of a Calabi-Yau manifold

- The SYZ conjecture predicts that at large ψ values, the loss of the real Monge-Ampere equation on the manifold will converge to the loss of the complex Monge-Ampere equation[1].
- Physics-informed neural networks (PINNs), which can incorporate losses that reflect physics into the network, such as real and complex Monge-Ampere loss, are very useful in solving for the Ricci-flat metric.
- The Monge-Ampère equation is: $J \wedge J \wedge J = \kappa\, \Omega \wedge \overline{\Omega}$
- The model's success in solving the complex M.A. equation (similarly defined for the real M.A. equation) is determined by sigma loss:

$$\sigma = \frac{1}{\mathrm{Vol}_{\mathrm{CY}}} \int_X \left\| 1 - \frac{1}{\kappa} \frac{J \wedge J \wedge J}{\Omega \wedge \overline{\Omega}} \right\|_1$$

Methods

- Neural networks have an input layer, output layer, and hidden layers.
- Multi-Layer Perceptrons (MLPs), a type of neural network, have activation functions on the nodes of hidden layers and learnable weights.
  - The performance of neural networks is influenced by hyperparameters, which include learning rate, width, and depth of the network.
  - Scanning for hyperparameters involves running the model for sets of hyperparameters to record the most efficient and accurate performance.

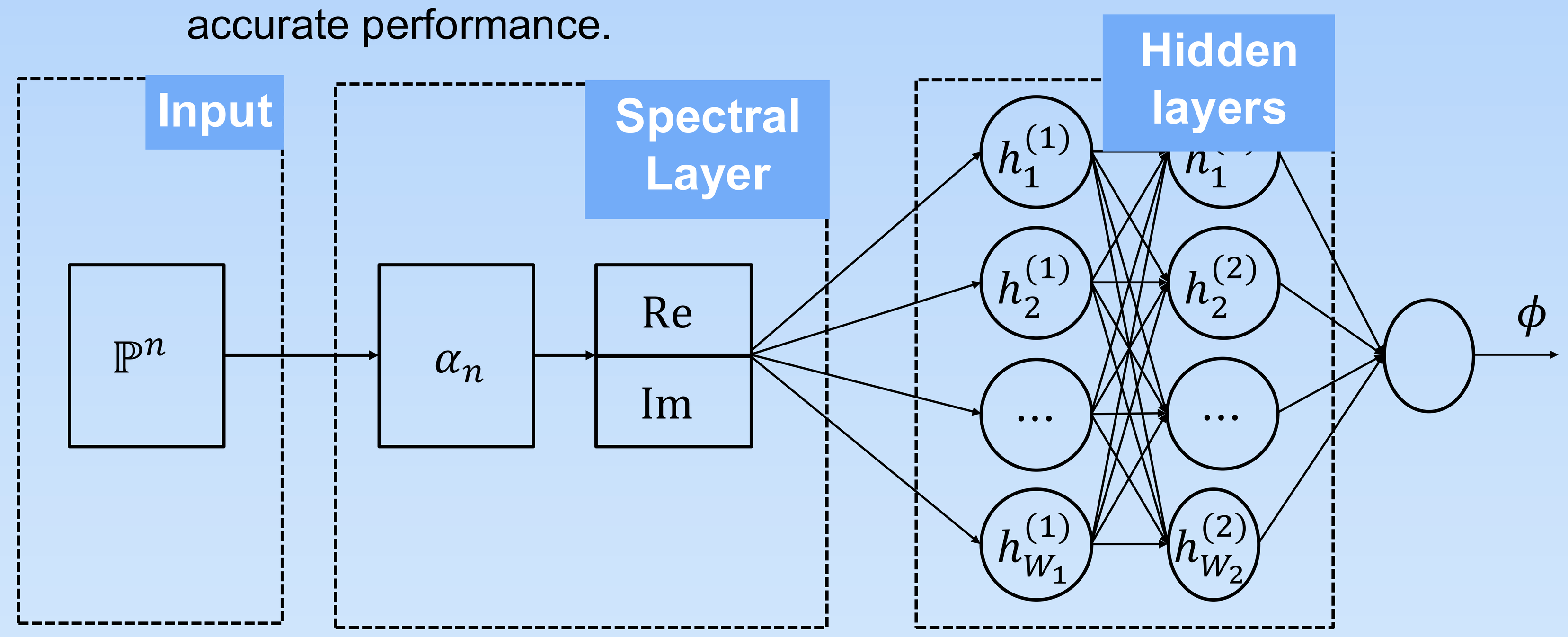

Figure 2: Spectral Neural Network

- Adaptive Fourier features can be used in addition to MLPs and have the potential to be more efficient and accurate, particularly when modeling very deformed CYs with large ψ values[2].
  - Adaptive Fourier features use sine and cosine modes to capture high frequency solutions of the metric on the CY manifold
  - Next, the adaptive Fourier features have a dense network applied to them to determine where the features are needed, because not all parts of the solution of the metric are high frequency, and applying the features everywhere on the manifold can cause overfitting..
- Using the previously generated repository cymyc [4], which is neural network code for finding the metric on CY manifolds, this project experimented with using adaptive Fourier features to solve for the metric of both the real and complex MA equations in addition to the TensorFlow dense neural network layers.
  - Once adaptive Fourier features are added to the TensorFlow Dense networks, the optimal hyperparameters for the Fourier features must be found.
  - Factors to be considered include how many layers the network should have, the width of those layers, and the number of features.
  - The loss values for the metric of the real and complex MA equations can then be analyzed over large ψ values to test the SYZ conjecture.

Results

- We tested the adaptive Fourier features across large values of ψ from 100 to 10000, recording the lowest loss for each value of ψ for both the real and complex MA equations.
- The results show that when ψ is 100, the loss for the real MA equation is much greater than the loss for the complex MA equation, but over time, their values converge. This is predicted by the SYZ conjecture, which states that the real MA loss should converge to the complex MA loss as ψ approaches infinity.

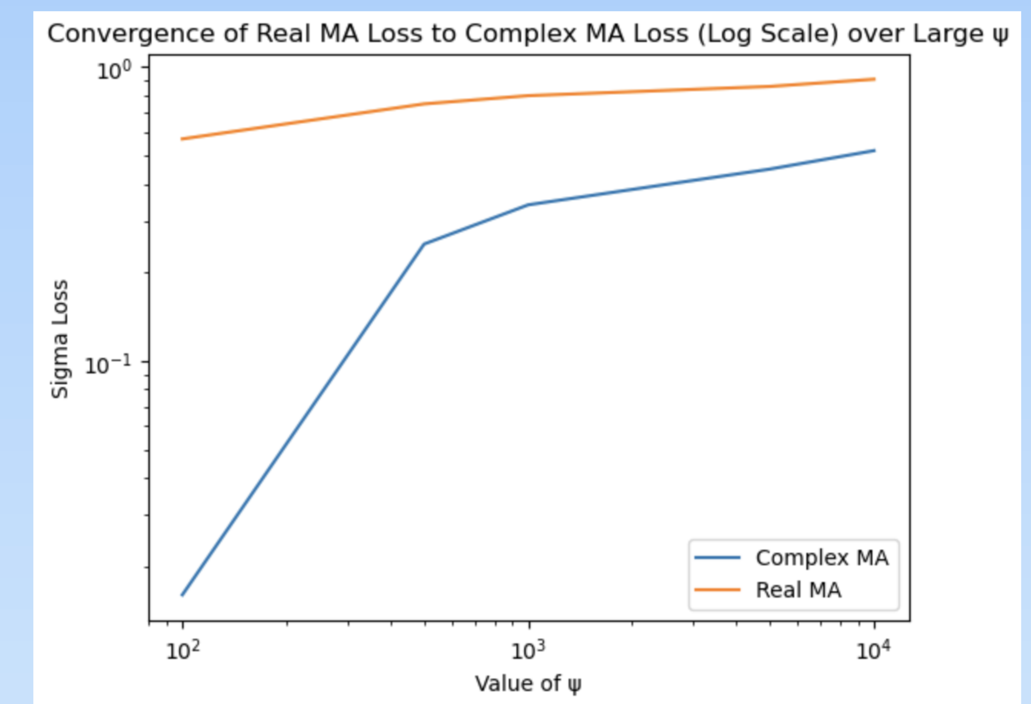

Figure 3: σ-losses for the complex MA (blue) and real MA (orange) equations as a function of ψ ∈ [100, 10000]. Both losses increase with ψ as the metric becomes harder to learn; the complex MA loss grows more steeply, reflecting the increasing geometric complexity near the large complex structure limit.

- This is further illustrated by figure 4, demonstrating that the ratio of the real MA loss to the complex MA loss decreasing as ψ increases.

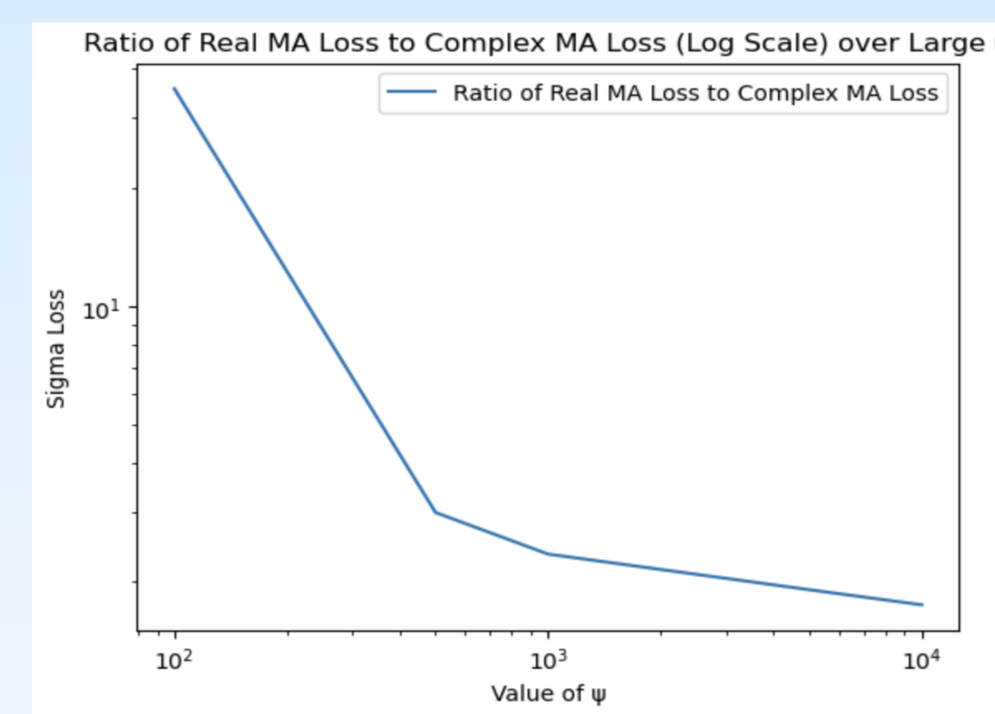

Figure 4: Ratio of real MA to complex MA σ-loss as a function of ψ. The decreasing ratio indicates that the complex MA loss grows faster than the real MA loss with increasing ψ, consistent with the SYZ picture in which the metric increasingly acquires real MA structure on the base as ψ → ∞.

References

[1] Yang Li, "SYZ Conjecture for Calabi-Yau Hypersurfaces," arXiv e-print {1912.02360}.
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[3] Per Berglund et al. "Machine Learned Calabi-Yau Metrics and Curvature," Advances in Theoretical and Mathematical Physics 27 (2023) 1107–1158.
[4] Giorgi Butbaia et al "cymyc: Calabi-Yau Metrics, Yukawas, and Curvature", Journal of High Energy Physics (2025) 1029-8479.

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Acknowledgements: Many thanks to the Hamel Center for URA funding.