Wave Modes for Current Drive in Magnetic Confinement Fusion

Torsten M. Diesel (torsten.diesel@unh.edu), ***Dr. Benjamin Chandran (advisor)***

URC ISE Poster #16700

University of New Hampshire

Introduction

- For a **plasma** (*fully ionized gas*) containing equal mixture of Deuterium and Tritium, the **Lawson criterion** (the power generated by fusion reactions exceeds the power lost due to heat transfer to the walls and to radiation) given by:

$$nT\tau_E > 3 \times 10^{21} \frac{\text{keV s}}{\text{m}^3}$$

- It cannot be said with confidence that the goal of economically sustainable fusion is possible, but exciting progress has been made in the past decade.
- Every time we (plasma physicists) believe we understand how plasma works, nature shows us new and exciting ways in which we're wrong!

Current Drive in Tokamaks

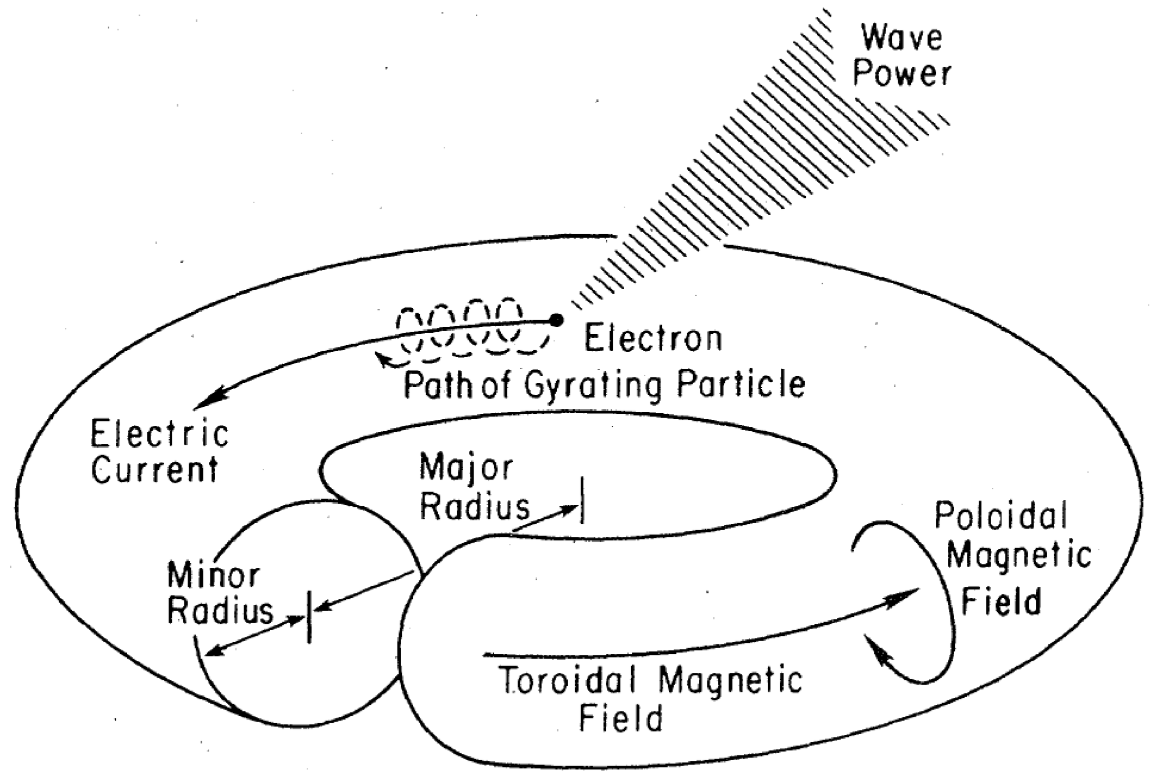


Figure 1: Tokamak Schematic. The **toroidal** magnetic field is easy to create with a normal cable running through the center of the toroid. The **poloidal** magnetic field cannot be induced the same way. [1]

- Poloidal field used to contain particles which would otherwise drift vertically out of the torus.

> "How does a twist in the field lines stabilize particle motion? The stabilization here is somewhat analogous to the stabilization against gravity of coal particles in a coal slurry in a long pipe by superimposing on the lonitudinal flow a swirling (poloidal) motion about the pipe axis." [1]

- I am interested specifically in the **wave-driven current drive** and in the wave modes involved.

Notations and assumptions

- Tensor quantities will be notated with an underline (such as $\underline{\chi}$)
- First order quantities vary with $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$
- The **cyclotron frequency** (frequency that particles of species s gyrate about magnetic field lines) is $\Omega_s \equiv q_s B_0 / m_s c$
- The **plasma frequency** (frequency that plasma responds to charge imbalances) is $\omega_{ps}^2 \equiv 4\pi n_s q_s^2 / m_s$

The Dielectric Tensor

The dielectric tensor, $\underline{\epsilon}$, is what we will be building towards.

$$\underline{\epsilon}(\omega, \mathbf{k}) = \mathbf{1} + \sum_s \underline{\chi}_s(\omega, \mathbf{k}) \qquad \text{where } \mathbf{j}_s = \underline{\sigma}_s \cdot \mathbf{E} = -\frac{i\omega}{4\pi}\underline{\chi}_s \cdot \mathbf{E}$$

It generalizes the dielectric constant in Maxwell's equations to anisotropic media, such as a plasma with a magnetic field running down the z-axis.

Jell-O Model

- Vast number of particles in a fully-ionized gas, model required.
- Choosing $T = 0$ allows us to replicate many known plasma wave modes (Alfvén, Langmuir-Tonks, whistlers, cyclotron waves, etc.).
  - The plasma can be thought of here as a blob of Jell-O, where particles are free to wobble about in place as they are influenced by electromagnetic forces.
- Equation of motion for particles of type s:

$$n_s m_s \frac{d\mathbf{v}_s}{dt} = n_s m_s \left(\frac{\partial \mathbf{v}_s}{\partial t} + \mathbf{v}_s \cdot \nabla \mathbf{v}_s\right) = n_s q_s \left(\mathbf{E} + \frac{\mathbf{v}_s}{c} \times \mathbf{B}\right) - \nabla \cdot \underline{\mathbf{\Phi}}_s$$

- $\underline{\mathbf{\Phi}}$ is the fluid stress tensor, which we can set to zero by the zero-temperature assumption.
- Linearizing the equations lets us simplify quantities like $\mathbf{B} \approx \mathbf{B}_0 + \mathbf{B}_1 + \ldots$
  - We also assume that n_s and $\mathbf{B}_0 = \hat{\mathbf{z}}B_0$ are finite, static in time, and spatially uniform. $\mathbf{v}_s$, $\mathbf{j}_s$, and $\mathbf{E}$ are all zero to zeroth order.
- Notable limit of this model: as no particles can be in motion, no current can occur here. v_0 would need to be nonzero, at least for electrons, for a current to flow.

How do waves travel in this homogenous, simplified model?

Parameterizing the problem

- The important quantities are the ratios of the plasma frequencies:

$$\alpha = \frac{\omega_{pe}^2}{\omega^2} \qquad \beta = \frac{\Omega_i}{\omega} \qquad \mu = \frac{m_i}{Zm_e} = \left|\frac{\Omega_e}{\Omega_i}\right| = \frac{\omega_{pe}^2}{\omega_{pi}^2}$$

- Wave polarizations can be represented as dimensionless parameters: "Right" and "Left" polarization, and "Plasma":

$$R, L = 1 \mp \frac{\alpha}{\mu\beta \pm \mu} \pm \frac{\alpha}{\mu\beta \mp 1} \qquad P = 1 - \alpha/\mu - \alpha$$

- The dielectric tensor may then be represented as:

$$\underline{\epsilon} \cdot \mathbf{E} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

$$S \equiv \frac{1}{2}(R+L) \text{ 'Sum'}, \qquad D \equiv \frac{1}{2}(R-L) \text{ 'Difference'}$$

$$\text{Wave equation: } \mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \underline{\epsilon} \cdot \mathbf{E} = 0$$

Where $\mathbf{n}$ is the refractive index of the plasma medium at that angle. [2]

Solutions

Nontrivial solutions of this wave equation are when the determinant of the following equation is zero:

$$\mathbf{n} \times (\mathbf{n} \times \mathbf{E}) + \underline{\epsilon} \cdot \mathbf{E} = \begin{pmatrix} S - n^2\cos^2\theta & -iD & n^2\cos\theta\sin\theta \\ iD & S - n^2 & 0 \\ n^2\cos\theta\sin\theta & 0 & P - n^2\sin^2\theta \end{pmatrix} = 0$$

We can represent this as a "quadratic" equation with :

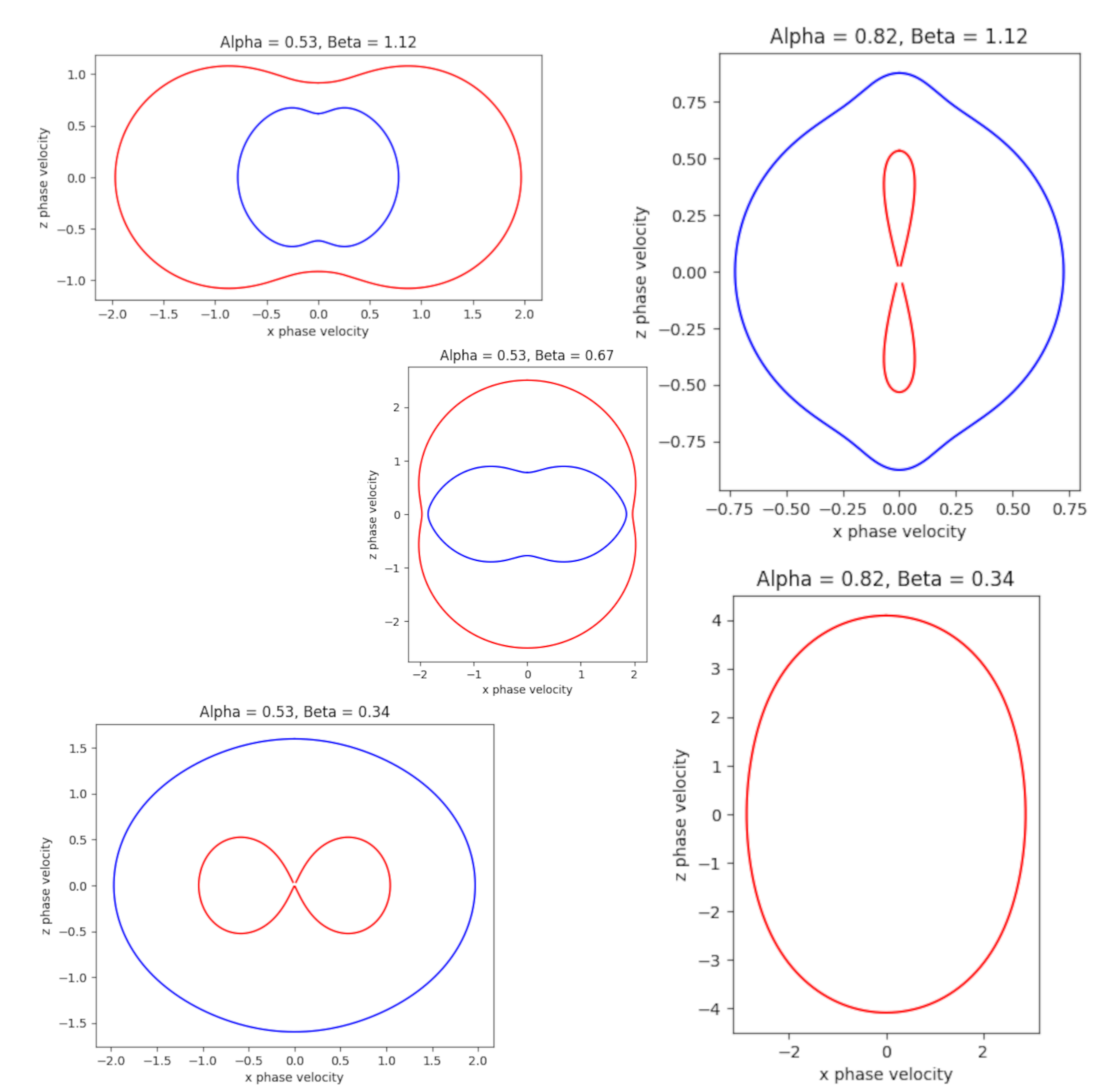
$$An^4 - Bn^2 + C = Cu^4 - Bu^2 + A = 0$$

$$A = S\sin^2\theta + P\cos^2\theta$$
$$B = RL\sin^2\theta + PS(1 + \cos^2\theta) \qquad u = \omega/kc = 1/n$$
$$C = PRL$$

What can we learn about this before plotting the solution?

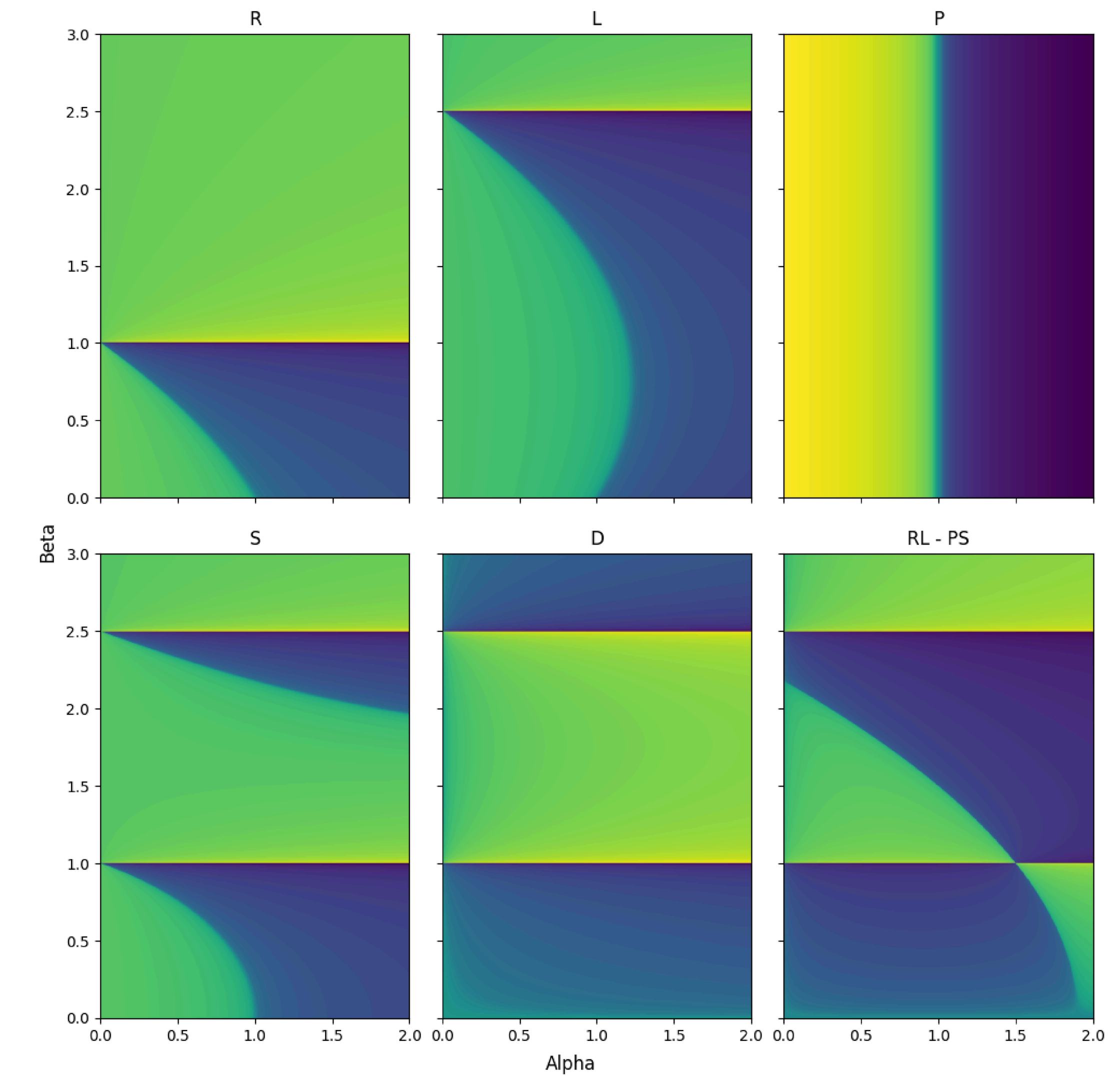
- Without changing the plasma parameters, $u(\theta)$ is a function of only θ.
- For each value of theta, we can get either zero, one, or two positive roots.
- Because θ appears either as $\cos^2\theta$ or $\sin^2\theta$, if $u(\theta)$ is a solution, then so is $u(-\theta), u(\pi - \theta), u(\theta - \pi)$.

Plotting Wave Normal Surfaces



- These plots are created by plotting the roots of $u(\theta)$ as functions of theta as a polar plot on the xz plane.
- For a given plasma with parameters α and β, what is the wave phase velocity at angle θ with respect to the direction of the magnetic field?
  - That is the velocity of the "wave normal surface", and the question these plots answer!
- Sometimes, a change in parameters completely changes the topological shape of the wave-normal surface, and sometimes it doesn't. Where are the boundaries?

The CMA Diagram



- These plots are in a $\alpha\beta$ phase space now.
  - Green areas represent that the quantity is positive, blue represents negative values.
- Shape of wave-normal surface is topologically changed when α or β crosses a "bounding surface".
- Thus, many different "distinct" plasma waves are really special cases of this generalized analysis.

Conclusions, notes

- Jell-o model doesn't hold when $T \neq 0$. Plasma dynamics then can be modelled with the Vlasov-Maxwell equations:

$$\frac{\partial f}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{d\mathbf{p}}{dt} \cdot \frac{\partial f}{\partial \mathbf{p}} = 0 \text{ ; Four Maxwell's Equations}$$

- That said, even though $\underline{\epsilon}$ is more complex (pun intended!), the steps are similar:
  1. starting with a distribution function $f(t, \mathbf{x}, \mathbf{v})$...
  2. find the currents for each species $\mathbf{j}_s$...
  3. which can be converted into suseptibilities $\underline{\chi}_s$...
  4. which add to the dielectric tensor $\underline{\epsilon}$...
  5. which then can be analyzed to find wave modes and a dispersion relation.
- Future research: the Bernstein wave is a specific wave mode which occurs in hot plasmas, and is a promising candidate for current drive. That work is still in progress.

Bibliography

[1] N. J. Fisch, "Theory of current drive in plasmas." Reviews of Modern Physics, Vol. 59, No. 1, 1987.

[2] T. H. Stix, *Waves in Plasma*. Springer, 1992.

[3] R. B. White, *The Theory of Toroidally Confined Plasmas*, 3rd ed. Imperial College Press, 2014.

[4] S. Glasstone, *Controlled Thermonuclear Reactions: An Introduction to Theory and Experiment*. 1975.