Asymptotic Methods of Angular Quantization in Free Boson CFT

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Motivation & Goal

Standard string theory has two types of strings:

- Closed strings
- Open strings

The process of *angular quantization* allows us to describe a new type of strings—called **stretched strings**.

These stretched strings are:

- Fundamentally different from open or closed strings
- Useful for studying black hole microstates
  - Major testing ground for quantum gravity

By studying the compact free boson in 2d Conformal Field Theory (CFT) in the context of angular quantization, we can better understand these stretched strings.

Our **goal** is to:

- Quantize the theory of the free boson $X(\tau,\sigma)$ and have it satisfy the **asymptotic conditions**:

$$\sigma \to -\infty: \quad \partial_\sigma X \to -\frac{i\alpha'}{R}n_1, \quad \partial_\tau X \to w_1 R \qquad (1)$$

$$\sigma \to \infty: \quad \partial_\sigma X \to \frac{i\alpha'}{R}n_2, \quad \partial_\tau X \to -w_2 R \qquad (2)$$

- These asymptotic conditions interest us since they inject momentum and winding into the string.

In order to do satisfy these conditions, we must

- Find mode expansion for the bosonic field $X(\tau,\sigma)$
- Compute the commutators
- Find a classical background satisfying these conditions

Background

The free boson we are studing is compact, and thus periodic, defined on a circle of radius R:

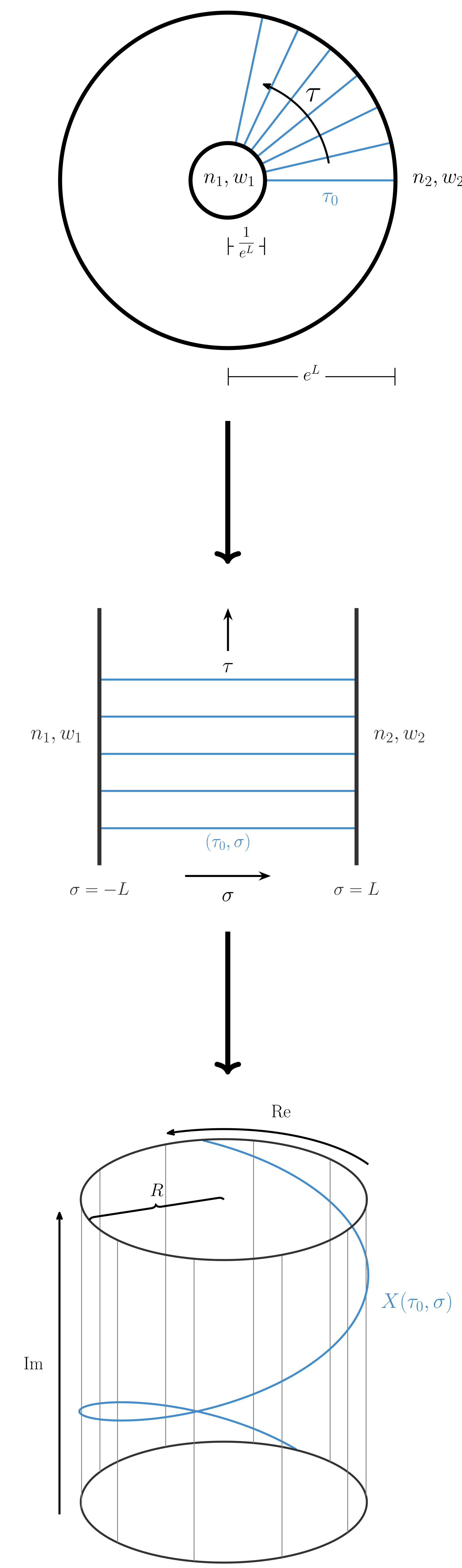
- $X \sim X + 2\pi R$

The bosonic field $X(\tau,\sigma)$ is:

- A map from the *worldsheet* to 1d *target spacetime*
  - The worldsheet has coordinates $\tau \in \mathbb{R}, \sigma \in [-L, L]$
  - τ is the time coordinate and σ is the space coordinate

In order to describe the correct physics of our system, we eventually take $L \to \infty$.

Procedure Visualization



Pictured above: First, we foliate the punctured disk in constant angle radially directed slices. Next, we map to the infinite strip, and as $L \to \infty$, we recover the full theory. Lastly, we can visualize the boson as the complexified string on the complex cylinder.

Technical Process & Results

Our asymptotic conditions motivate the action

$$S = \frac{1}{2\pi}\int d\tau d\sigma \Big\{ \frac{1}{2\alpha'}\left[(\partial_\tau X)^2 + (\partial_\sigma X)^2\right] + \left[\frac{-in_1}{R}X + \frac{aL}{\alpha'}(\partial_\tau X - w_1 R)^2\right]\delta(\sigma + L) + \left[\frac{-in_2}{R}X + \frac{aL}{\alpha'}(\partial_\tau X + w_2 R)^2\right]\delta(\sigma - L)\Big\},$$

The most general solution to the equation of motion and the boundary conditions gives the **mode expansion** for $X(\tau,\sigma)$:

$$X(\tau,\sigma) = x + \frac{\pi i\alpha' p}{L(1+2a)}\tau - \frac{i\alpha'}{2R}\left[(n_1 - n_2)\sigma + \left(\frac{n_1+n_2}{2L(1+2a)}\right)(\tau^2 - \sigma^2)\right] + \sqrt{2\alpha'}\sum_{\ell\in S^+}\left[\sin\left(\frac{\pi\ell(\sigma+L)}{2L}\right) - \frac{1}{\pi a\ell}\cos\left(\frac{\pi\ell(\sigma+L)}{2L}\right)\right]\left(\frac{\alpha_\ell e^{\frac{-\pi\ell\tau}{2L}} + \alpha_{-\ell}e^{\frac{\pi\ell\tau}{2L}}}{\ell}\right),$$

where S^+ is the set of positive solutions to the transcendental equation $(\pi a\ell)^2 - 1 = 2\pi a\ell\cot(\pi\ell)$.

Upon quantization, we obtain the **commutators**

$$[x,p] = i, \quad [x,\alpha_\ell] = 0 = [p,\alpha_\ell], \quad [\alpha_\ell, \alpha_{\ell'}] = \ell\left(1 + \frac{1+2a}{(\pi a\ell)^2}\right)^{-1}\delta_{\ell,\ell'},$$

and computed the classical Hamiltonian H

$$H = \frac{n_1+n_2}{2\pi i R}x + \frac{\pi\alpha'}{2L(1+2a)}p^2 + \frac{\alpha' L}{4\pi R^2}\left[\frac{1+3a}{3}\left(\frac{n_1+n_2}{1+2a}\right)^2 + \frac{(n_1-n_2)^2}{2}\right] + \frac{\pi}{2L}\sum_{\ell\in S^+}\left(1 + \frac{1+2a}{(\pi a\ell)^2}\right)\alpha_{-\ell}\alpha_\ell.$$

The **classical background** which obeys (1) & (2) is

$$X_{cl}(\tau,\sigma) = \tfrac{i\alpha'}{2R}\big((n_2 - n_1)\sigma + (n_1+n_2)|\sigma|\big) + \big(w_1\theta(-\sigma) - w_2\theta(\sigma)\big)R\tau.$$

The operators in $X(\tau,\sigma) - X_{cl}(\tau,\sigma)$ create and annihilate the particle states along the string which satisfy the asymptotic conditions.

Future Work

In the future, we would like to compute the mode expansions of all the operators in the theory, then compute their correlators.

References

[1] Nicholas Agia and Daniel L. Jafferis. *Angular Quantization in CFT*. 2022. arXiv: 2204.11872 [hep-th]. URL: https://arxiv.org/abs/2204.11872.

[2] Nicholas Agia and Daniel L. Jafferis. *The 2d Free Boson Minkowski CFT with Asymptotic Charges*. 2024. arXiv: 2402.05167 [hep-th]. URL: https://arxiv.org/abs/2402.05167.