Nonperturbative Energy Corrections in Quantum Mechanics Due to Instantons

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Introduction

- Typical perturbation theory misses exponentially small energy corrections generated by tunneling effects, called instantons.
- Instantons can be understood through the exact WKB method or path integral methods
- The WKB method, although easier, does not extend to higher dimensional cases.
- Compare results given from the path integral method to those obtained with the WKB method.

Background Information

- Unitless Symmetric Double Well Potential

$$V(x) = \frac{1}{2}x^2(1-x)^2$$

We will be using g as the coupling constant of the two

- 1-Instanton Solution/tunneling solution to the Equation of Motion:

$$x_{cl}(\tau) = \frac{1}{1-e^{-(\tau-\tau_0)}}$$

where τ_0 is the zeromode shift of the solution.

- Path Integral Methods

Integrate all possible paths from a starting point to an end point. Decompose as a sum around "saddle points" i.e., solutions to the equation of motion.

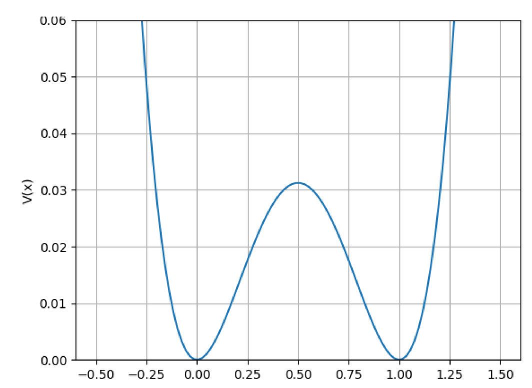
- Feynman Diagram loop calculations (pictured right)

Each loop diagram calculates a term in the expansion

- Energy correction

Without tunneling the ground state is degenerate; tunneling between minima causes an exponentially small energy split between the two ground states.

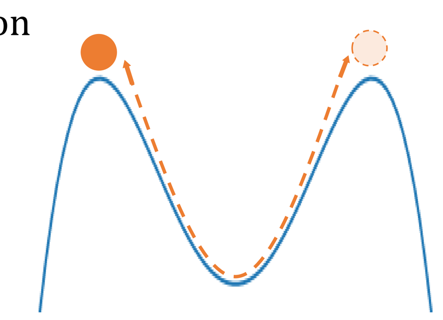
Figure 1: Symmetric Double Well Potential



The distance between the two minima $\propto \frac{1}{\sqrt{g}}$.

The height of the potential $\propto \frac{1}{g}$.

Figure 2: A visual representation of an instanton



A toy model representing tunneling effects as a ball rolling from one peak of the inverted potential to another

Green Functions (Feynman Propagators)

For free Space:

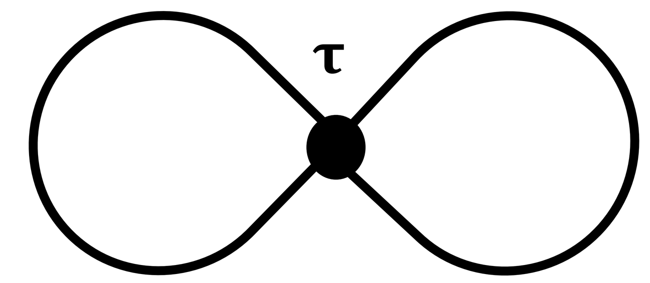
$$\therefore G_{free}(\tau,\tau') = \frac{1}{2}e^{-|\tau-\tau'|}$$

For One Instanton Background:

$$\therefore G(\tau,\tau') = \begin{cases} \dfrac{g[e^{3\tau+\tau'}+8e^{2\tau+\tau'}+e^{\tau}(2e^{\tau'}(3\tau-3\tau'-4)+8+e^{-\tau})]}{2(1+e^{\tau})^2(1+e^{\tau'})^2}, & \tau<\tau' \\ \dfrac{g[e^{\tau+\tau'}(e^{2\tau'}-2(3\tau-3\tau'+4)+8e^{\tau'})+8e^{\tau'}+e^{-\tau+\tau'}]}{2(1+e^{\tau})^2(1+e^{\tau'})^2}, & \tau\geq\tau' \end{cases}$$

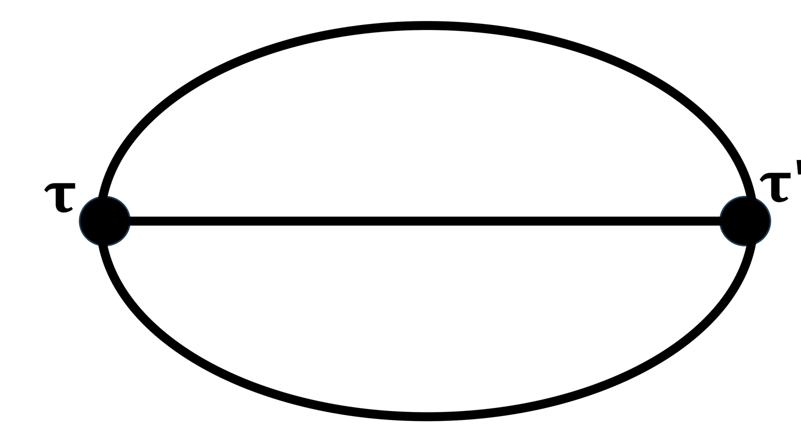
Feynman Diagrams For Energy Corrections

Figure-Eight Diagram



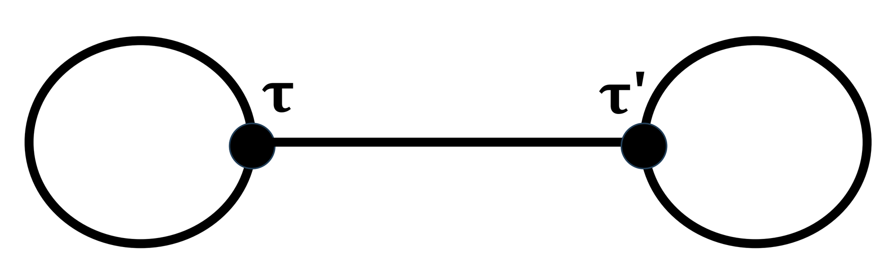
$$\int_{-\infty}^{\infty} d\tau \left[\frac{1}{8}\left(\frac{-4!}{g}\right)G(\tau,\tau)^2\right]$$

Sunrise Diagram



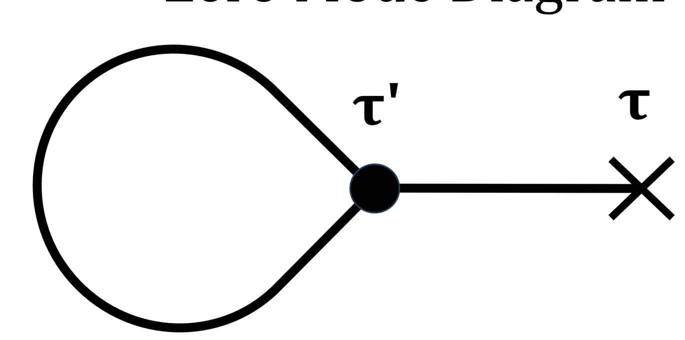
$$\int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau \left[\frac{1}{12}\left(\frac{-3!}{g}\right)(2x_{cl}(\tau)-1)(2x_{cl}(\tau')-1)G(\tau,\tau')^3\right]$$

Dumbbell Diagram



$$\int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau \left[\frac{1}{8}\left(\frac{-3!}{g}\right)(2x_{cl}(\tau)-1)(2x_{cl}(\tau')-1)G(\tau,\tau)G(\tau,\tau')G(\tau',\tau')\right]$$

Zero Mode Diagram



$$\int_{-\infty}^{\infty} d\tau' \int_{-\infty}^{\infty} d\tau \left[\frac{1}{2}\left(\frac{-\ddot{x}_{cl}(\tau)}{|\dot{x}_{cl}(\tau)|}\right)\left(\frac{-3!}{g}\right)(2x_{cl}(\tau')-1)G(\tau,\tau')G(\tau',\tau')\right]$$

Results

- Figure-Eight: $-\frac{3gT}{8} - \frac{97g}{280}$
- Sunrise: $\frac{gT}{4} - \frac{53g}{210}$
- Dumbbell: $\frac{9gT}{8} - \frac{117g}{280}$
- Zero Mode : $-\frac{49g}{10}$
- Free Background: -gT
- Total energy correction: $-\frac{71g}{12}$

Conclusions

- We found the ground state energy splitting to be:

$$E_{0,-} - E_{0,+} \approx \frac{2}{\sqrt{\pi g}}e^{\frac{-1}{6g}}\left(1-\frac{71g}{12}\right)$$

- The above matches with the results in the literature obtained by the WKB method

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References

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