Robust Stabilization at Uncertain Equilibrium Point by Output Derivative Feedback Control

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Problem

Uncertainty in the knowledge of equilibrium states presents a challenge for analysis and control methods.

Goal

Design output derivative feedback controller to stabilize uncertain system at true equilibrium.

Hypothesis

Robustness in controller design allows for stablization of wide range of uncertainties in the equilibrium.

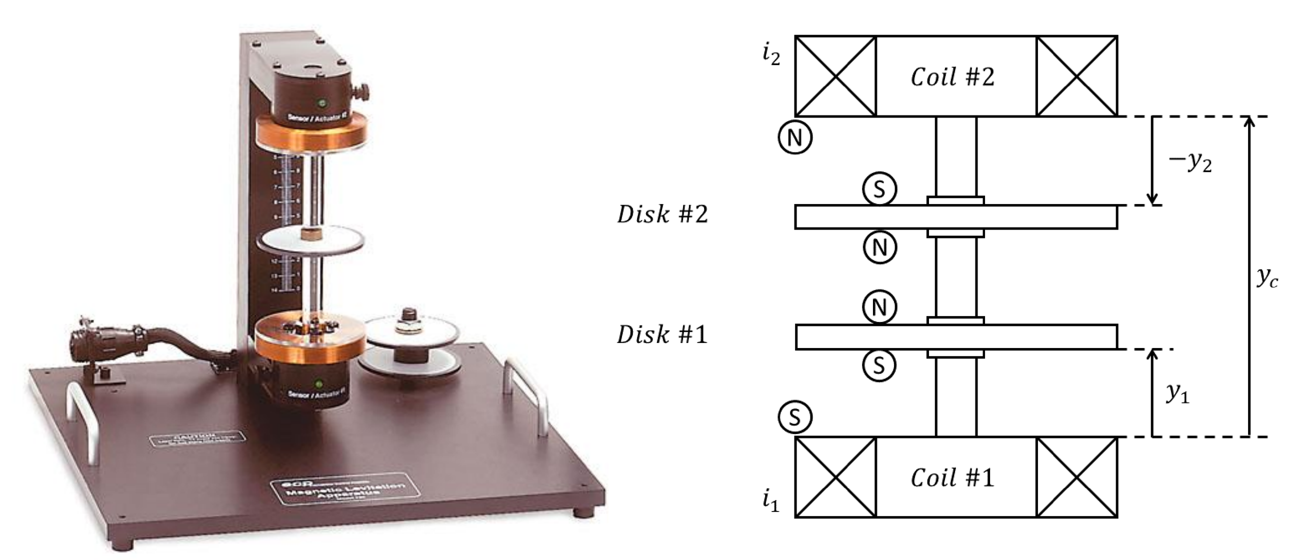
Introduction

- Equilibrium states represent natural resting points of dynamic systems.
- Locating the equilibrium states of complex systems difficult in practical applications due to
  - the complexity of the system dynamics,
  - from chaotic behaviors exhibited by the system due to variations in external operation conditions.
- Knowledge of equilibrium state is essential for system analysis and control techniques.
- Controlled system may be stabilized away from its true resting state leading to wastage of limited control resources for systems with constrained actuators.
- Norm-bounded uncertainties are considered in the robust controller design.

Mathematical Formulation

- Nonlinear dynamic system in the state space form
  - $\dot{x} = f(x,u)$,
  - $y = Cx$.
- Linearized system about nominal equilibrium $\hat{x}_e$
  - $\delta\dot{x} = (A_n + A_\Delta)\delta x + (B_n + B_\Delta)u$,
  - $\delta y = (C_n + C_\Delta)\delta x$.
- Dynamic Output Derivative Feedback Controller
  - $u = K\zeta$,
  - $\dot{\zeta} = A_c\zeta + B_c\dot{y}$.
- x = state vector, u = input, y = output, x_e = true equilibrium.

Experimental Setup

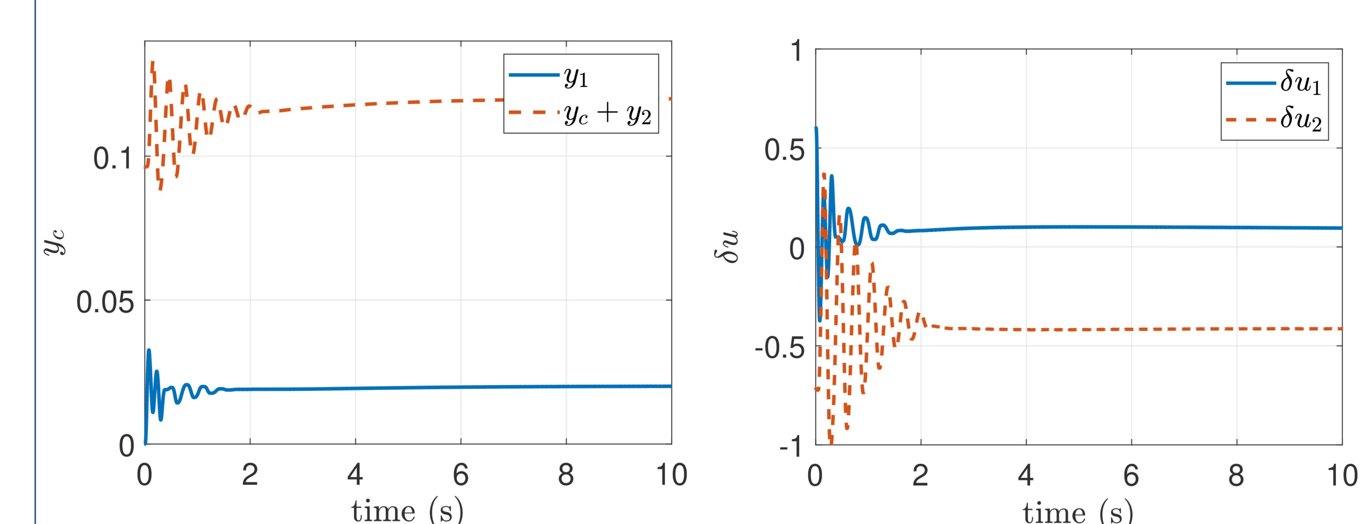


2-Disk Magnetic Levitation System

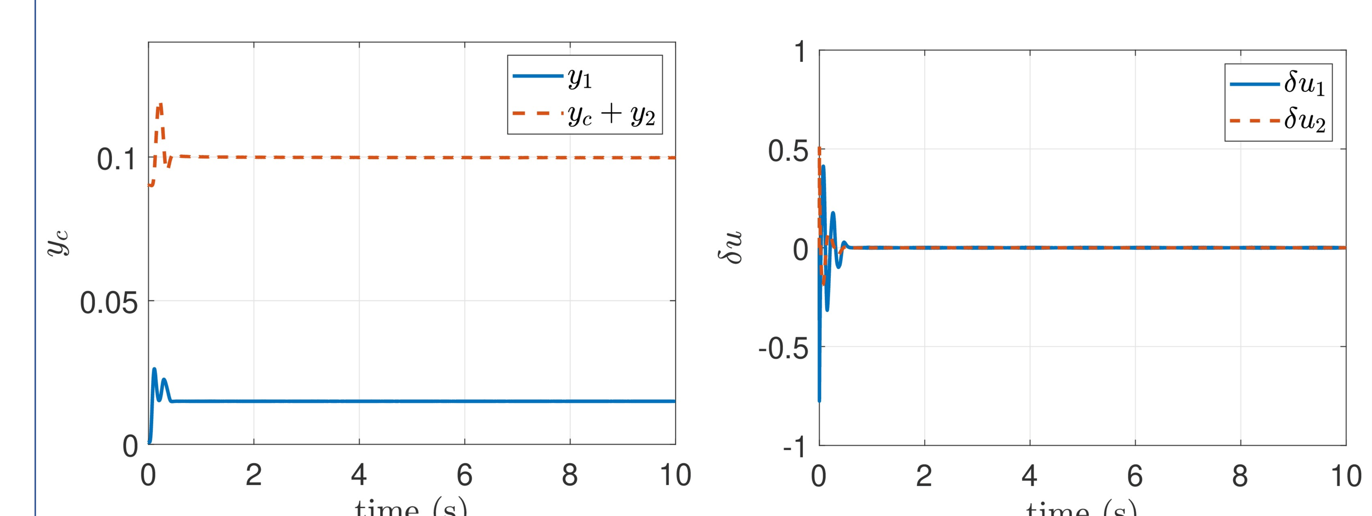
- MagLevs:
  - electromagnetic forces used to levitate a moving body.
  - precise knowledge of magnetic equilibrium position is crucial but challenging.
- MagLev test setup consists of two magnetic disks controlled by actuator coils at the top and at the bottom.
- **System model:**
  - $m\ddot{y}_1 + c_1\dot{y}_1 + F_{m_{12}} = F_{u_{11}} - F_{u_{12}} - mg$,
  - $m\ddot{y}_2 + c_2\dot{y}_2 - F_{m_{12}} = F_{u_{22}} - F_{u_{21}} - mg$.
  - m = mass of magnets, g = gravity, c_i = damping coefficient of magnet i, $F_{m_{12}}$ = magnetic force between magnets 1 & 2, $F_{u_{ij}}$ = force from coil i acting on magnet j.

Results

- Equilibrium point of MagLev derived from linearized MagLev system model at nominal disk location $y_1 = y_2 = 0.02cm$.
- Comparison between Proportional Integral (PI) controller ($Fig.1$) and Robust Output Derivative Feedback (RODF) controller ($Fig\ 2.$).



$Fig.1$ MagLev under PI Controlller



$Fig.2$ MagLev under RODF Controller

Conclusion

- PI controller:
  - Magnetic disks stabilized near the nominal location, but the control effort δu in $Fig.1$ does not approach 0. Disk positions do not correspond to true equilibrium of MagLev. The control offset is consumed to maintain system away from true equilibrium.
- RODF controller:
  - Magnetic disks stabilized at $y_1 = 0.015cm$, $y_2 = 0.04cm$. However, control effort $\delta u = 0$ which indicates natural resting position of disks. Controller stabilizes MagLev at true equilibrium. Control resources are not wasted to maintain disks at non-equilibrium point.