On Machine Learning and the Phase of the S-Matrix

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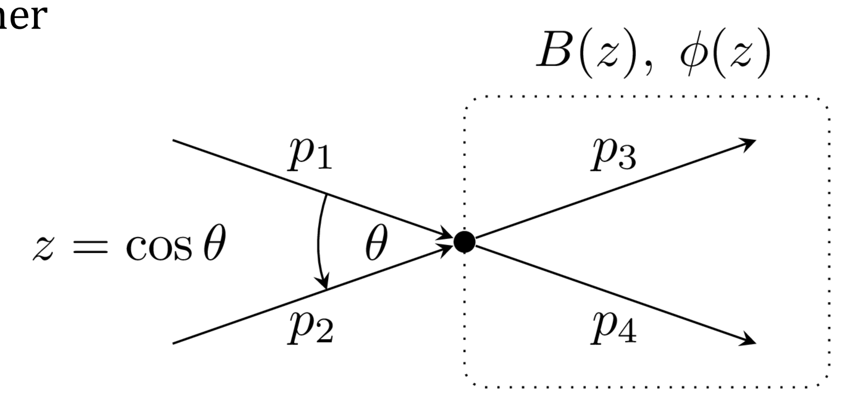
Introduction

The Scattering Matrix (S-Matrix) is a matrix representation of the probability amplitudes of particles scattering off each other

- The S-Matrix is a fundamental part of Quantum Field Theory (QFT)
- The S-Matrix contains all possible ways that particles can scatter off each other



- In the 1960's D. Atkinson solved families of solutions with nontrivial phase shift ambiguities
- Atkinson solved for a parameter called sinμ, which determines if solutions have ambiguous phase shifts

$$K(z) \equiv \int_{-1}^{1} dz_1 \int_{0}^{2\pi} d\phi_1 \frac{B(z_1)B(z_2)}{4\pi B(z)}$$

$$\sin\phi(z) = \int_{-1}^{1} dz_1 \int_{0}^{2\pi} d\phi_1 \frac{B(z_1)B(z_2)}{4\pi B(z)} \cos[\phi(z_1) - \phi(z_2)]$$

$$\sin\mu = \max_{-1\le z\le 1} K(z), \ \ |\sin\phi(z)| \le \sin\mu$$

- Atkinson solved for a lowest sinμ value of 2.15, which remained the low until 2023
- In 2023 Dersy, Schwartz, and Zhiboedov published a paper using machine learning to solve for a new lowest sinμ value of 1.67
- Dersy et al. published their S-Matrix Bootstrap program with their paper
- We have modified the S-Matrix Bootstrap program to attempt to obtain a better result for sinμ
- Our modifications were increasing the number of points used to evaluate the sinμ integral from 25 to 100, and changing the network to have an Adaptive Fourier Layer
- We have been able to reproduce Dersy et al.'s results with these modifications, along with possibly finding a new low for sinμ

Adaptive Fourier Features

- Adaptive Fourier Features decompose the input data into a mixture of high frequency modes and the original, allowing for the machine to learn additional latent patterns
- This is implemented by taking the sine of the data at six different frequencies, and training some neurons on that modulated data instead, before combining again with the original to go through further processing
- We have also analyzed the loss function, integrand, and other hyperparameters in search of a new lower bound for sinμ

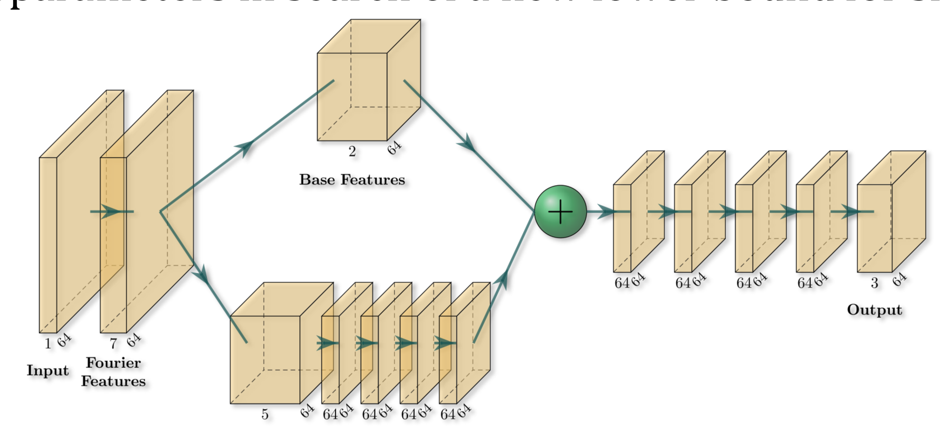


Figure 1: *How data flows through the neural network*

Point Loss Comparison

Original

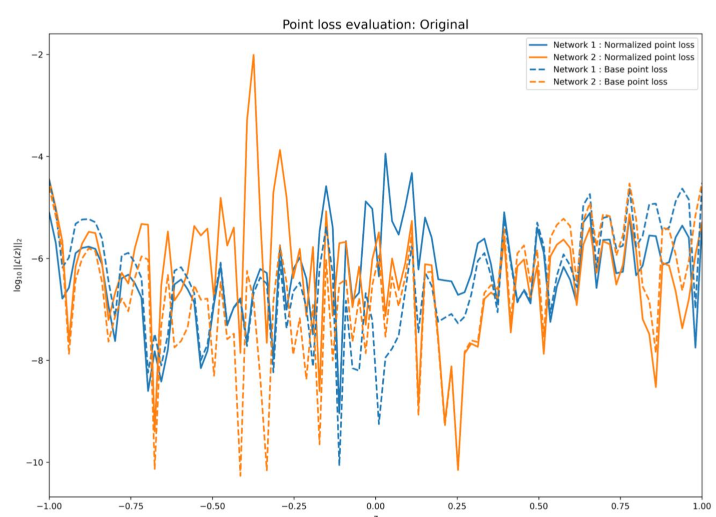


Figure 2: *Point Loss from Dersy, Schwartz, and Zhiboedov.*

Adaptive Fourier

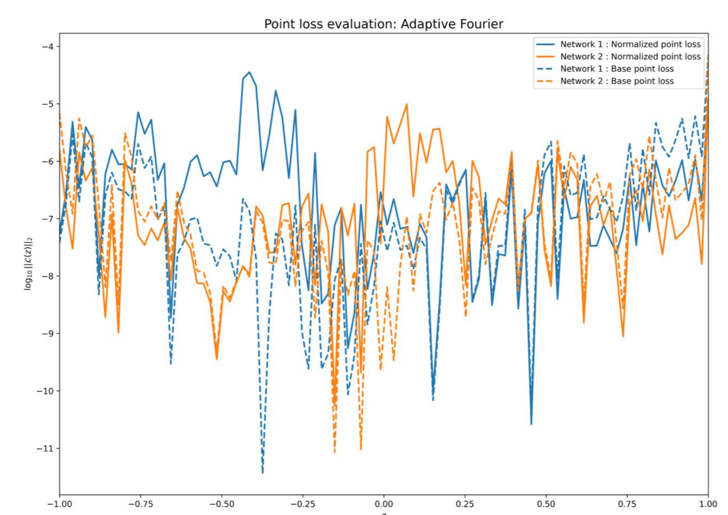


Figure 3: *Point loss with Adaptive Fourier Features.*

Additional Integration

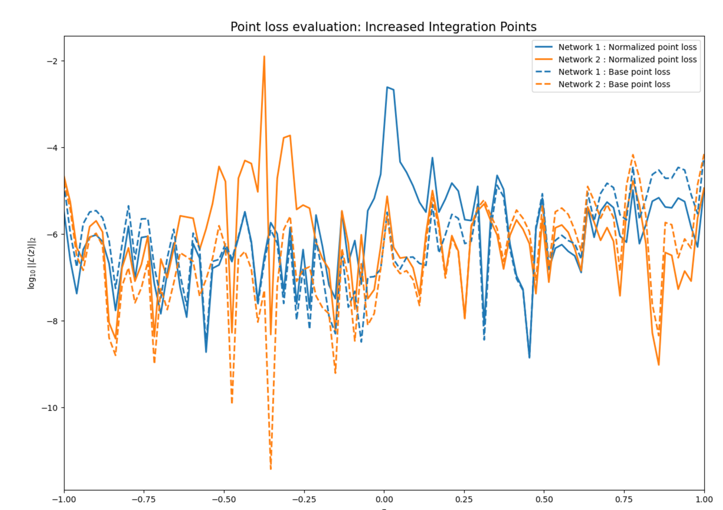


Figure 4: *Point loss with Additional Integration points.*

Loss Landscape Comparison

Original Results

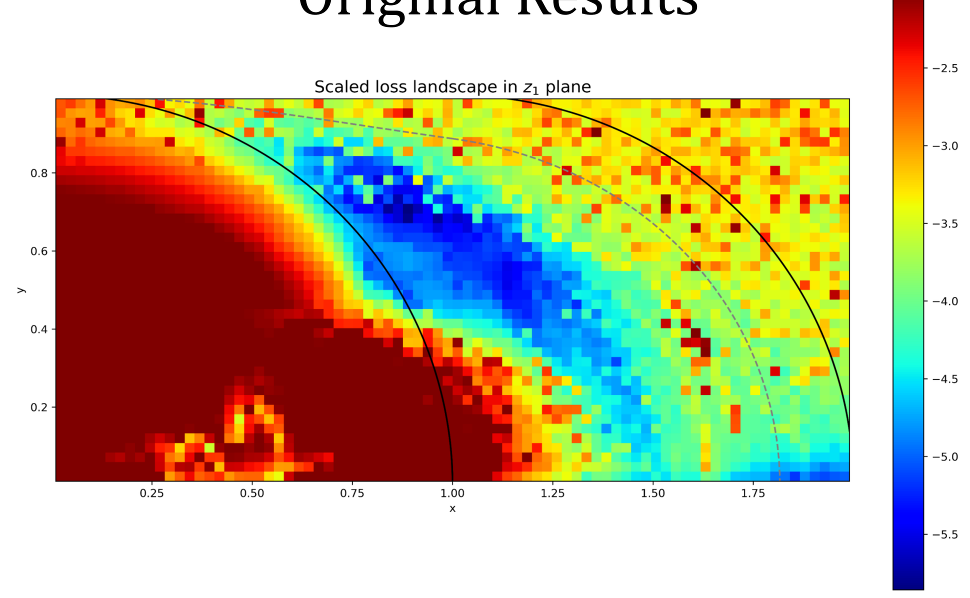


Figure 5: *Original Dersy, Schwartz, and Zhiboedov Loss Landscape.*

Adaptive Fourier Features

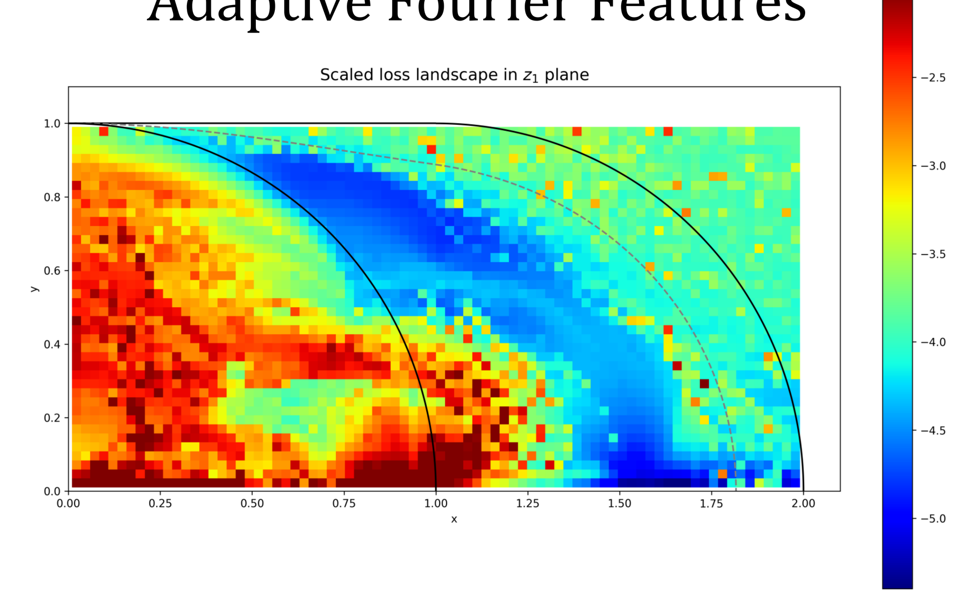


Figure 6: *Loss Landscape Using Adaptive Fourier Features.*

sin(μ) Graphs

Machine Learning Atkinson's Solution

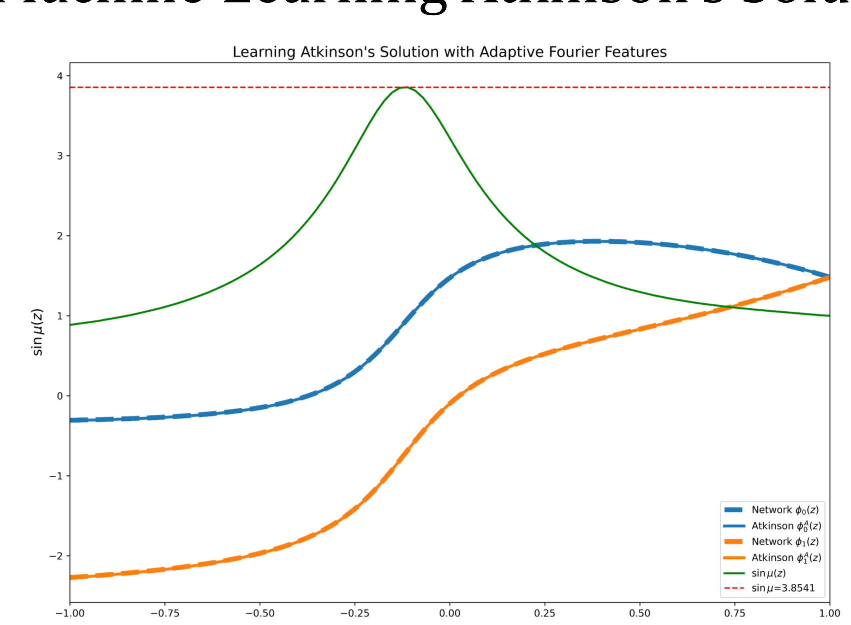


Figure 8: *Reproducing Atkinson's Solution (solid blue/orange) with Machine Learning (dotted)*

Potential New Lower Bound

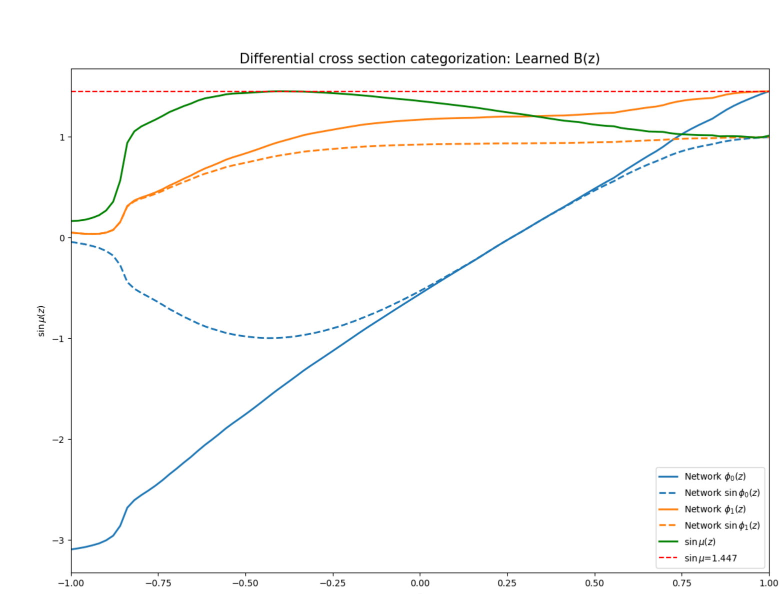


Figure 9: *Summary plot for a possible new minimum sinμ.*

Data & Results

We have been able to reproduce and improve upon the results of Dersy et al. with different settings (see figures 2-6).

- One comparison is the difference between the sinΦ(z) and the integral
- High losses indicate that the network has not learned the possible phase amplitude well
- Dersy et al.'s results for loss were mostly in the range 10^{-4} to 10^{-6}, with some of the order 10^{-2} and 10^{-10}
- Using an Adaptive Fourier layer most results are of the order 10^{-6} or lower, with the highest loss being of order 10^{-4}
- Using 100 integration points instead of 25 the loss distribution follows the same general patterns as the original, but with some high losses being more pronounced
- With Adaptive Fourier layers, loss is generally better

The sinμ value of 3.86 solved for by Atkinson for the set of zeros (1.2, 0.6) will be considered the "true" result to compare our results against.

- The original S-Matrix Bootstrap program produced sinμ = 3.839
- The S-Matrix Bootstrap with 100 integration points produced sinμ = 3.8611
- The S-Matrix Bootstrap with an Adaptive Fourier Layer produced sinμ = 3.8554

Conclusions

Both the increased number of integration points and the addition of Adaptive Fourier Features were able to reproduce and improve upon the results of the paper.

- Increased integration points produced a value closest to Atkinson's, but had high computation time and the same issue in the loss
- Adaptive Fourier Features produced a value much closer to Atkinson's than the original code with the same number of integration points, and had a more consistent loss
- A new lowest sinμ value of 1.447 (Figure 9) has possibly been discovered, though further analysis of the integral error and loss function are required to verify this

Future of the project

- Try different integration methods for solving sinμ
- Try different loss functions

Acknowledgements

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References

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