EMPC-Based Flight Controller Design for a Quadrotor with Unbalanced Payload

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Systems Design Project of Advanced Control Lab

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Introduction

Model predictive control (**MPC**) is widely recognized for its effectiveness in control, but its computational complexity for solving the online optimization problem, particularly for fast systems such as flight control, poses a challenge. To address this, we introduce an innovative inner-outer loop control structure that incorporates explicit MPC (**EMPC**) which can transfer the overall optimization process offline. Additionally, integral sliding mode control (**ISMC**) is integrated to mitigate the effects of unbalanced payloads.

Dynamics & Control Diagram

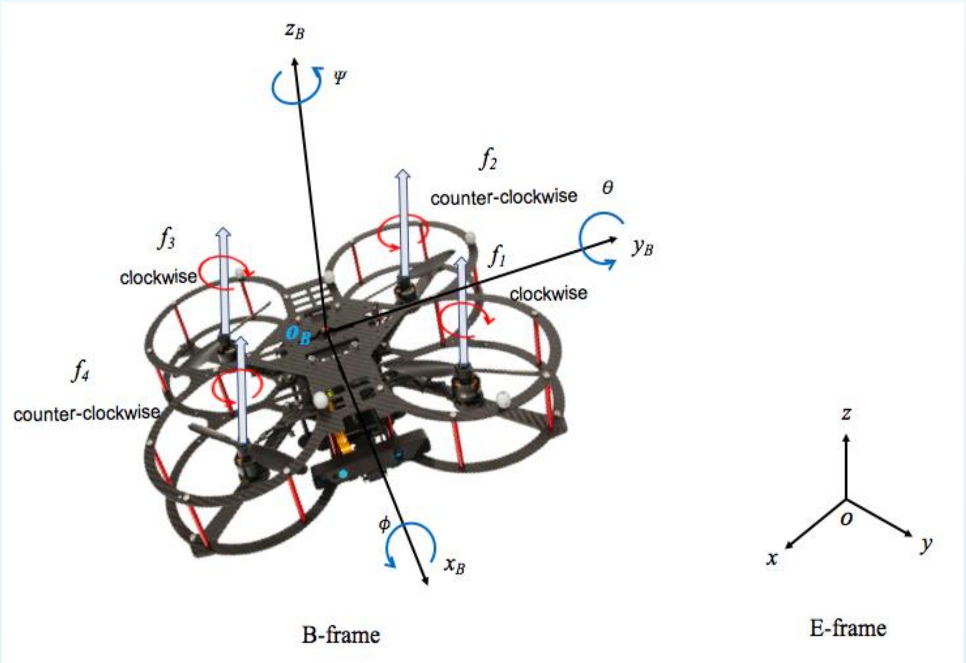

Fig. 1. The coordinate systems of the quadrotor.

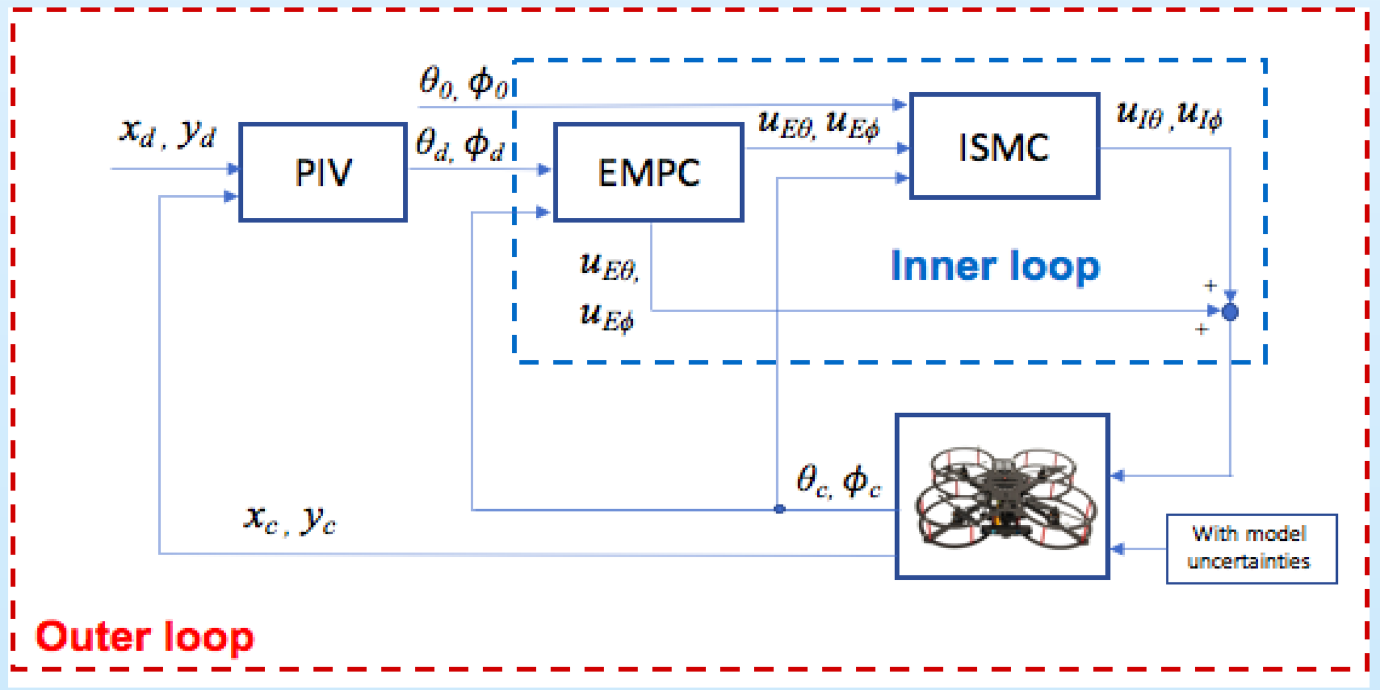

Fig. 2. A schematic diagram of the proposed EMPC-ISMC strategy.

Simplified and Linearized Dynamics of the quadrotor along x-axis:

$$\begin{bmatrix}\dot{x}\\ \ddot{x}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\end{bmatrix}+\begin{bmatrix}0\\ g\end{bmatrix}\theta,\quad \begin{bmatrix}\dot{\theta}\\ \ddot{\theta}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\end{bmatrix}+\begin{bmatrix}0\\ \frac{1}{I_{YY}}\end{bmatrix}\tau_\theta.$$

From equations above, we can observe the relationships between positions x, angles θ and toques τ in the quadrotor dynamics.

Methods

MPC problem is defined by

- Objective that is minimized, e.g., distance from the origin, sum of squared/absolute errors, etc.
- Internal System Model to predict system behaviors, e.g., linear, nonlinear, stochastic, etc.
- Constraints that have to be satisfied, e.g., on inputs, outputs, states, linear, quadratic, etc.

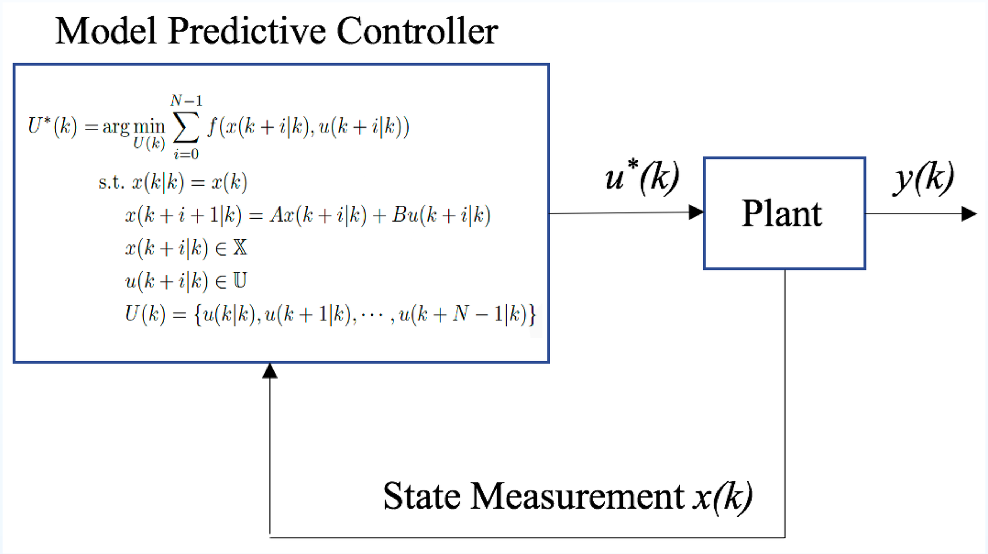

Fig. 3. General formulation and workflow of MPC.

Explicit MPC provides an alternative that does not require solving optimization problems online.

- Pre-compute control law u^*_N as function of state x so that online computation is dramatically reduced.
- Main Tool: Multiparametric Programming.

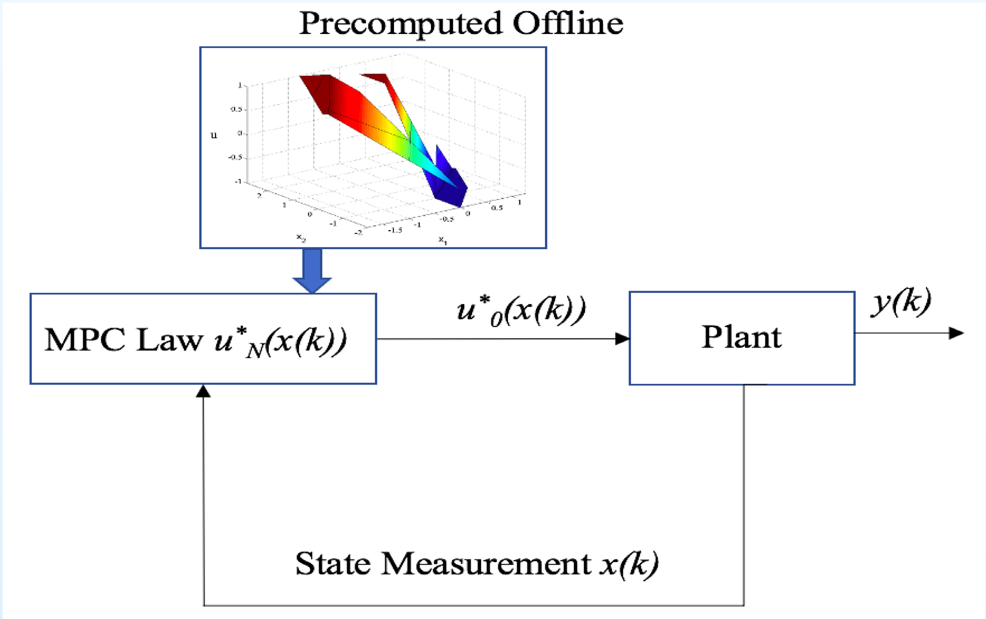

Fig. 4. General formulation and workflow of EMPC.

ISMC is a robust control method for handling system nonlinearities, uncertainties and external disturbances. It uses a sliding surface based on system states and desired output to guide the system. The controller, typically comprising proportional and integral terms, stabilizes the system and maintains sliding along the surface.

$$\dot{\tilde{X}}_\theta(t) = A_\theta \tilde{X}_\theta(t) + B_\theta \tau_\theta(t) + M_\theta \xi_{\tilde{X}}(t) + f_{\tilde{X}}(t)$$

where M_θ is known, and $M_\theta = B_\theta D_\theta$, for some D_θ with appropriate dimensions. $\xi_{\tilde{X}}(t)$ is the unknown disturbance or model uncertainty with a known upper bound. $f_{\tilde{X}}(t)$ is the unmatched uncertainty with a known upper bound. The sliding surface is defined as $\sigma(t) = \tilde{G}\tilde{X}_\theta(t) - \tilde{G}\tilde{X}_\theta(0) - \tilde{G}\int_0^t (A_\theta X(\tau) + B_\theta u_E(\tau))\,d(\tau)$, where $\tilde{G} = \left(B_\theta^{\mathrm{T}} B_\theta\right)^{-1} B_\theta^{\mathrm{T}}$. By subtracting $\tilde{G}\tilde{X}_\theta(0)$, we have $\sigma(0) = 0$, and the reaching phase does not exist. The attitude tracking controller $u_\theta(t)$ is made up of two parts

$$u_\theta(t) = u_E(t) + u_I(t)$$

References

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Video

Experimental Results

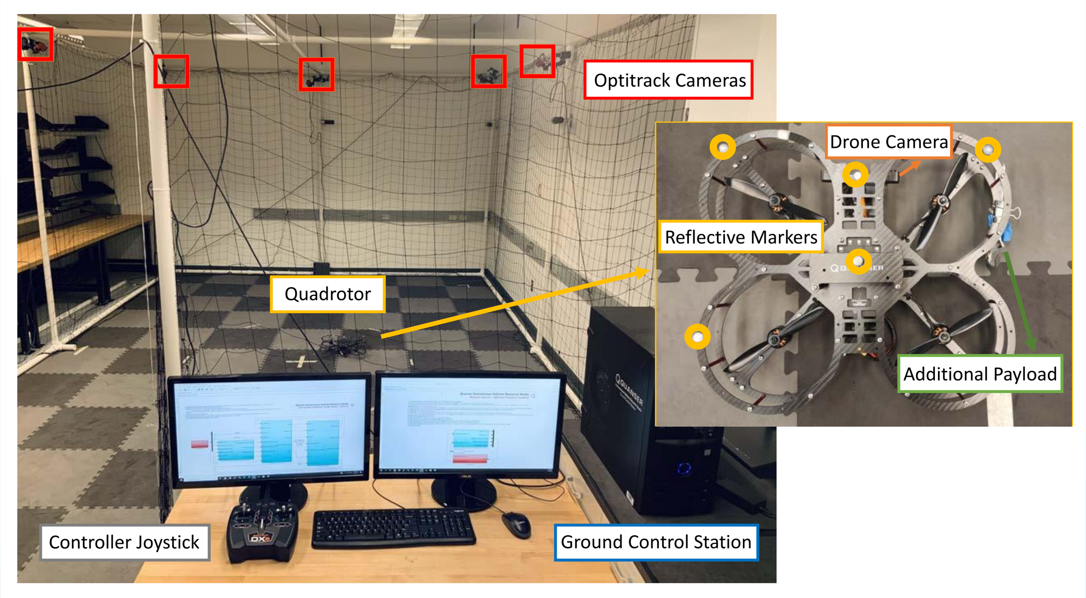

Fig. 5. Experimental setup of the quadrotors system.

Case studies for waypoints tracking and mean squared error (**MSE**) :

Case	Additional Weight	Controller
A	No	EMPC
B	Yes	EMPC
C	Yes	EMPC+ISMC

The MSEs of implementing the designed controllers

case	a	b	c
x	0.0069	0.0136	0.0064
y	0.0165	0.0217	0.0169
ϕ	0.0137	0.0021	0.0007
θ	0.0157	0.0264	0.0231

Desired Locations	Position 0	Position 1	Position 2	Position 3
[X, Y, Z] (m)	[0.00, 0.00, 0.50]	[-0.25, -0.50, 0.50]	[0.05, -0.10, 0.50]	[0.35, 0.30, 0.50]

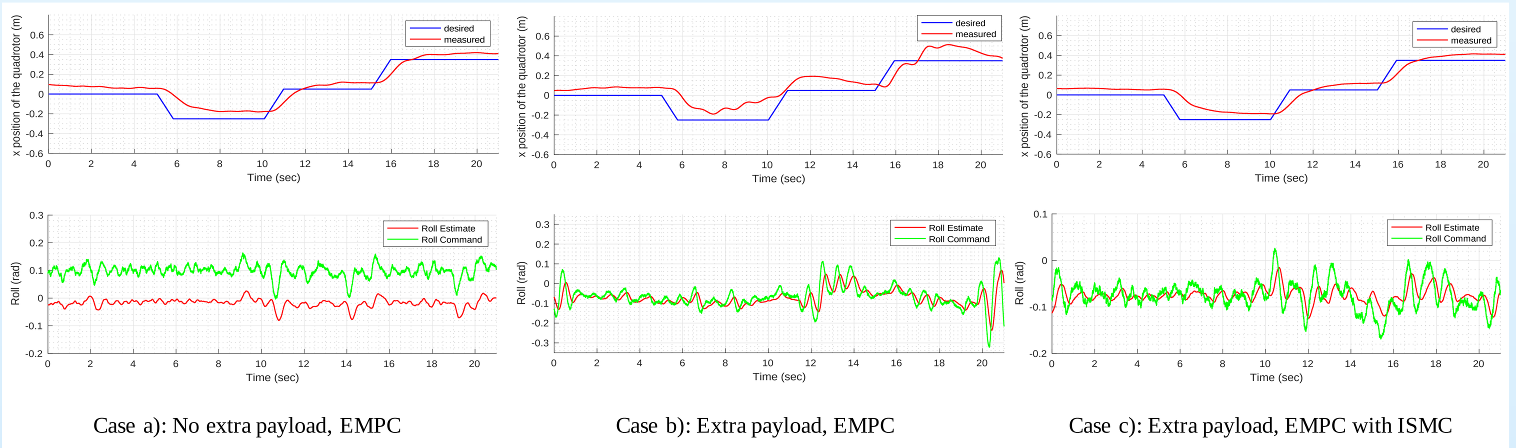

Fig. 6. Experimental results of case a), b), and c) for time response of x positions and φ angles.

Discussions for SDGs

Exploring the United Nations' 17 Sustainable Development Goals (**SDGs**), we have identified that our research aligns with the following global objectives:

Goal 9: Industry, Innovation, and Infrastructure. Our research contributes to advancements in technology and innovation by improving control strategies for quadrotors. This can enhance the infrastructure for aerial robotics, which has applications in various industries such as agriculture, transportation, and emergency response.

Goal 11: Sustainable Cities and Communities. Quadrotors equipped with efficient control algorithms can be utilized for urban monitoring, disaster management, and infrastructure inspection, contributing to the development of sustainable cities and resilient communities.

Goal 13: Climate Action. Quadrotors can be deployed for environmental monitoring and surveillance, aiding in climate research, deforestation monitoring, and disaster risk reduction efforts. Optimal control strategies, like **explicit MPC**, can improve their energy efficiency and reduce carbon emissions.