Should We *Transform* Our Understanding of Linearity in Generalized Linear Models?

David Heit[1], Mairi Poisson[1], Andrew Butler[1], Waldemar Ortiz-Calo[2], Robert Montgomery[3], Remington Moll[1]

1 Wildlife Modeling and Management Laboratory, University of New Hampshire, Durham, NH, USA **2** Wildlife Biology Program, W.A. Franke College of Forestry, University of Montana, Missoula, MT, USA
3 Department of Biology, University of Oxford, Oxford, UK

The problem with linearity

- Linear models, and by extension **generalized linear models (GLMs)** are an incredibly widespread and useful tool in wildlife ecology.
- Linear models inherently assume that the relationship between a predictor and a response is linear, which may not perfectly capture the **complex nature of ecological relationships**.
- Generalized linear models (GLMs) expand the applicability of linear models beyond just positive continuous data using link funcitons.
- However link functions do not remove the linearity assumption, the assumption shifts to the link scale.
- While the effects of non-linearity are more intuitive for simple linear regression, they become less straigt-forward in GLMs.

Link functions

IDENTITY

$y_i = \beta_0 + \beta_1 * x_i$

$y_i = \beta_0 + \beta_1 * \log(x_i)$

LOGIT

$\log(\text{odds ratio}_{y_i}) = \beta_0 + \beta_1 * x_i$

$\log(\text{odds ratio}_{y_i}) = \beta_0 + \beta_1 * x_i^2$

LOG

$\log(y_i) = \beta_0 + \beta_1 * x_i$

$\log(y_i) = \beta_0 + \beta_1 * x_i + \beta_2 * x_i^2$

linear on link scale
non-linear on link scale

CASE STUDY

Songbird Occupancy Modeling

POINT COUNT SURVEYS

- ***Study area:*** 63 sites across UNH owned woodlands and public areas in Durham and Dover, NH.
- ***Field Season:*** Summer breeding season 2021 & 2022
- ***Protocol:*** All surveys completed between 30 mins. before sunrise and 10:00am. Surveys last 8 minutes. Each site is surveyed 3 times per season.

	No. Obs.	Effect (linear)	Effect (logged)	DIC (linear)	DIC (logged)
Gray Catbird	52	0.178	-0.247	312.7	309.2
Nothern cardinal	17	1.131	0.6259	141.2	165.65
Wood Thrush	11	-0.48	-0.928	130.2	110.3
Black & White Warbler	15	-1.24	-1.5	134.99	108.7
House Sparrow	14	1.51	1.45	150.58	120.21

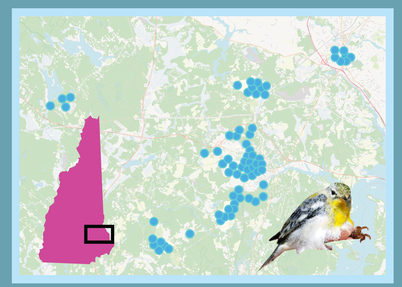


Figure 1. Survey sites for bird point count sampling in southeastern New Hampshire, USA.

MODELING METHODS

- Retained birds with >20 observations (42 spp.)
- Used bayesian occupancy-detection models to conduct Box-Tidwell linearity tests (Box & Tidwell 1962)

BOX - TIDWELL TEST (Box & Tidwell 1962)

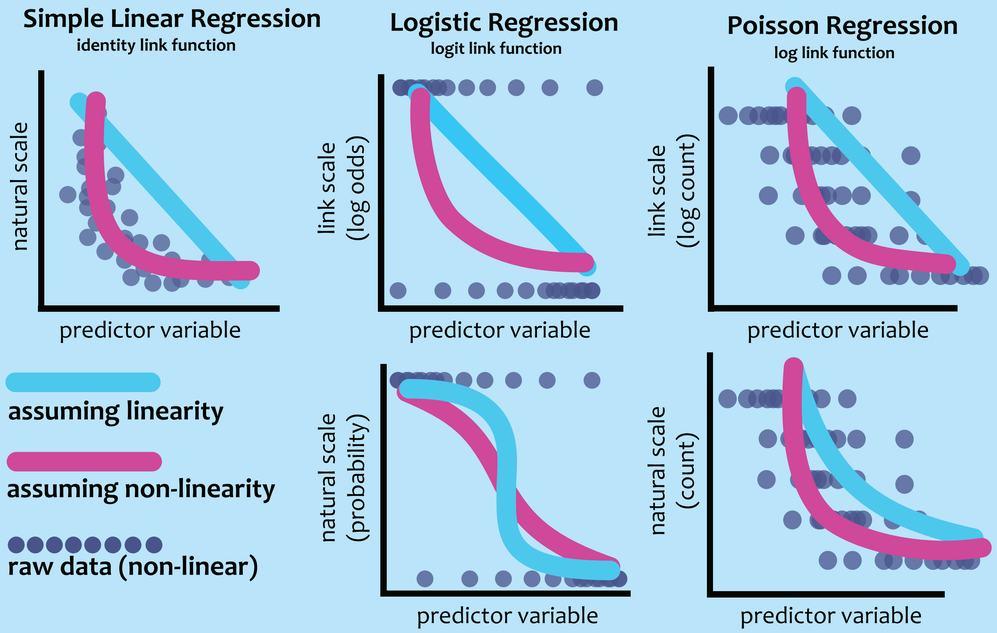
$\log(\text{odds ratio}_{y_i}) = \beta_0 + \beta_1 * x_i + \beta_2 * x_i * \log(x_i)$

RESULTS

- **12% (5/42) of species failed Box-Tidwell test**
- **Of those 5 species, 4 had improved DIC with log-transformed covariate.**
- **One species (Northern Cardinal) failed Box-Tidwell test, but the model was not improved by logging, indicating some other non-linear relationship**

DATA SIMULATION

Non-linearity in GLMs



FROM THE LITERATURE

How often are ecologists thinking about linearity?

LITERATURE REVIEW

273 ecology papers published in the last 5 years
Results shown here are preliminary (50 papers)

RESULTS

Tested for linearity:	**14%**
Transformed covariates:	**20%**
Fit non-linear model:	**22%**

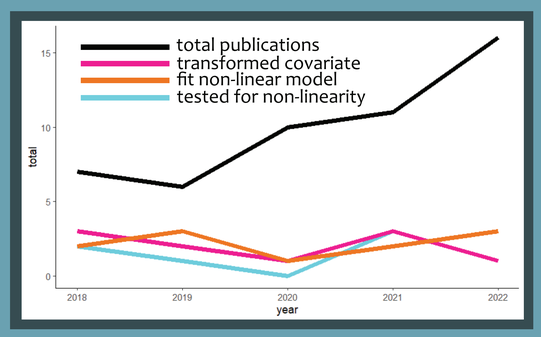


Figure 2. Results from preliminary literature review showing the relative frequency of incorporating efforts to address linearity.

Putting it all together

- Based on preliminary results of literature review, **testing for linearity is not common practice.**
- It is more common to fit a non-linear model than to test for linearity.
- Relationships between covariates and link-scale responses can be non-linear.
- Non-linearity can also **vary by species** within a community.
- Non-linear and linear relationships can vary in direction and magnitude within the same species.
- Log-linear transformation may not be sufficient to address non-linearity.
- Future best practices may include testing for linearity in communities, species, populations, and individuals.

Future Directions

- Expand literature review
- Add two more case studies to explore the dynamics of linearity in Poisson regression and generalized additive models (GAMs).
- How do species with non-linear effects affect community-level models?

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REFERENCES

Box G., Tidwell P. 1962. Transformation of the independent variables. *Technometrics*.