Developing a Fractal Model of Electrical Discharge Trees

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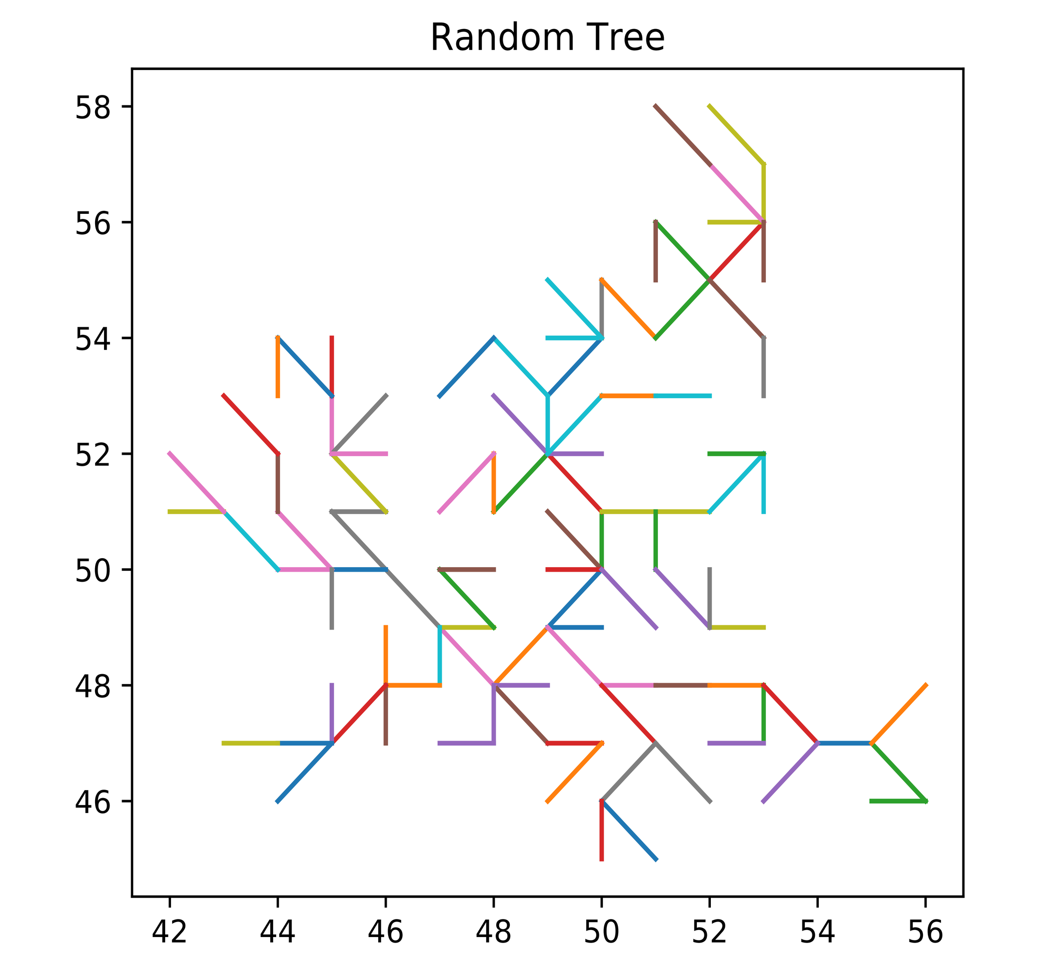
Previous Research

The goal of this project is to build on previous research to develop a model of electrical discharge trees (see example below). To begin, I have attempted to reproduce a model made by Pasko et Al. [2000]. His model is based on qualitative observations of electrical discharge trees. The electrical discharge tree is represented by a network on a square grid. In each step of the simulation a 'branch' connecting two points of the grid is added somewhere on the tree. The position of the branch is selected randomly and weighted by the strength of the electric field. The model is two dimensional and uses cylindrical coordinates, assuming azimuthal symmetry.

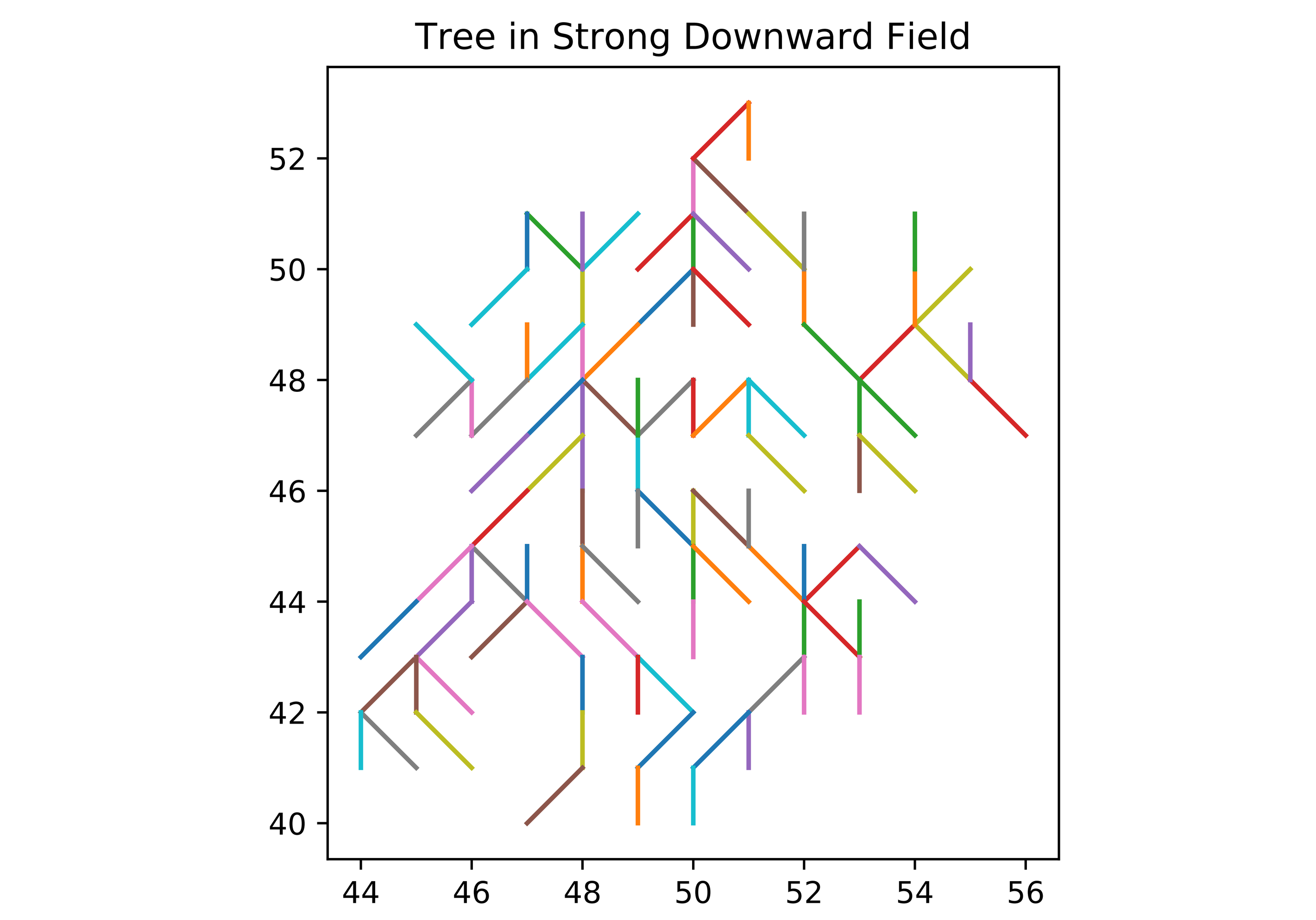


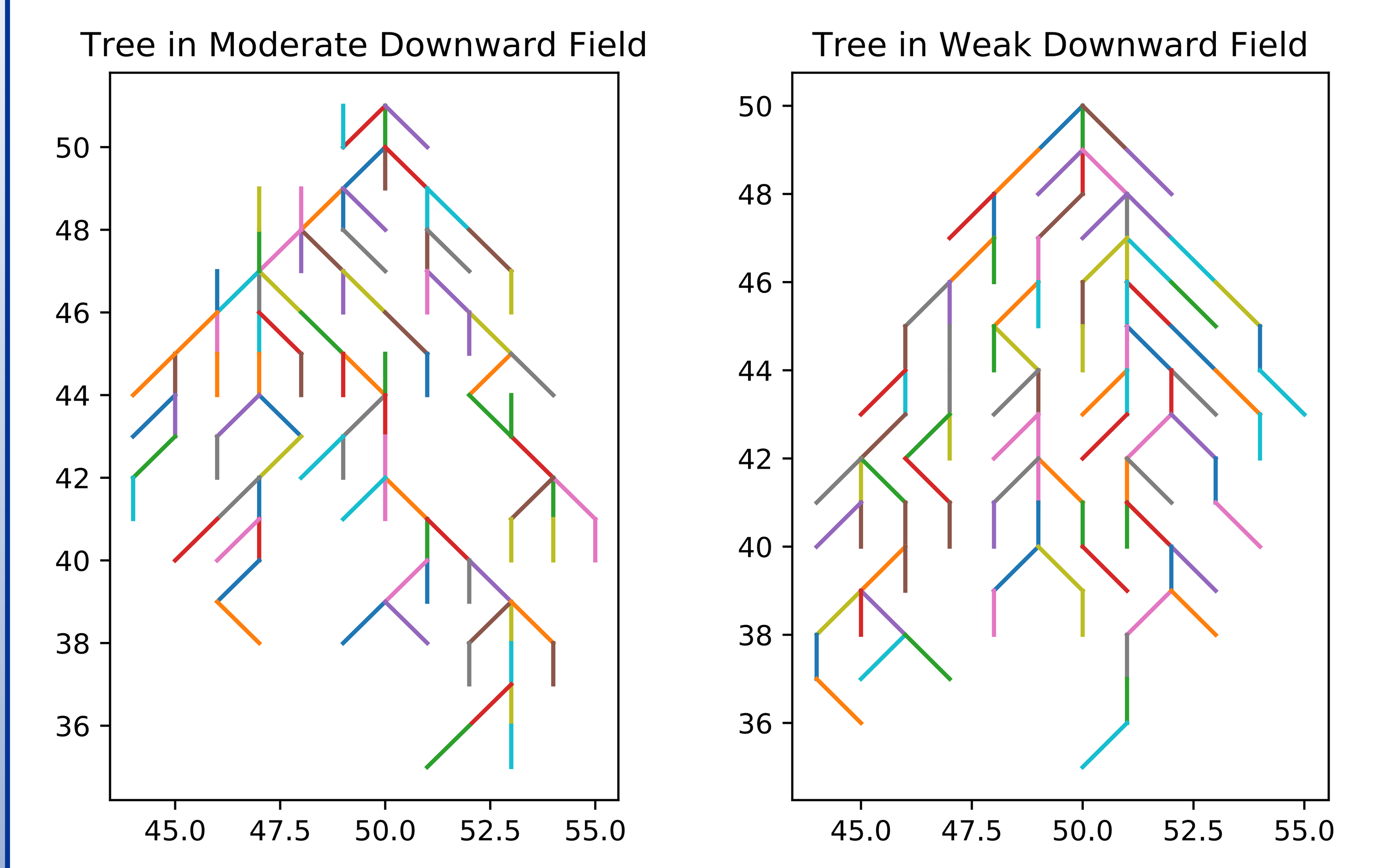
Random Tree Generator

The first step in reproducing Pasko's model is to make a random tree generator. First the model iterates over each node of the tree. For each node of the tree, each neighboring point is also iterated over. Each pair of points constitutes a potential branch. However, not every potential branch is a valid candidate, so some must be pruned from the list. For example, branches which cross existing branches are not considered. Of the valid candidates, one is selected randomly and added to the tree. The process is then repeated with the updated tree until the tree almost reaches the boundary. An example of such a random tree is shown above.

Adding Physics

The random tree model creates something that vaguely resembles an electrical discharge tree, but only insofar as it is a jagged tree structure. To make the model more realistic, the probability of adding each branch is weighted by the local electric field. At this stage, we don't worry about the contribution to the electric field due to the tree itself (that comes later). In addition, we only want branches to be added if the electric field is strong enough. Streamers, which are the filaments that make up electrical discharge trees, come in two varieties, positive and negative, which propagate with and against the electric field, respectively. The critical field necessary for the propagation of positive and negative streamers is determined empirically and is higher for negative streamers than for positive ones. I am using the values recommended by Pasko et Al. [2000]. At this stage, the electric field is hard wired into the code. Each candidate is a assigned a weight proportionate to the difference between the local electric field and the critical electric field. Examples of trees generated with three strengths of electric field are shown below and to the right.





Conclusion and Future Work

So far the model has been developed to the point that it can create random non-intersecting trees, which can be biased according to an external field. We see that the stronger the field is, the more common negative (upwards) steamers become. In addition, we see that all horizontal streamers, which are normal to the electric field are eliminated. We also see that despite vertical streamers being more highly weighted than diagonal streamers by a factor of radical two, diagonal streamers are more common as there are two diagonal candidates for every horizontal candidate.

The only work remaining needed to reproduce Pasko's model is calculating the electric field due to the tree itself, which entails solving Laplace's Equation with the tree as a boundary condition. Further plans to expand the model include moving from an azimuthally symmetric model to a fully three dimensional model. We also want to break free of the grid which puts an arbitrary constraint on the length of the streamer section, which requires solving Poisson's equations by a different method.

Works Cited

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