Shear Instabilities of Tidal Currents in Inlets and Estuaries

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Hypothesis

Tidal currents in narrow inlets and channels can have horizontal velocity gradients that produce instabilities in the flow (so-called shear waves) that can lead to the spinoff of large eddies. The resulting vortices, or eddies, may impact navigation, transport of organic or inorganic matter (i.e. larvae, oil spills, etc.), and cause mixing of momentum in coastal estuaries.

Introduction

The along-channel tidal current speed may vary both temporally and spatially across the channel, or inlet, as a function of the tides causing a horizontal velocity gradient (Figure 1). The cross-channel current shear provides a background vorticity field that supports a linear instability mechanism. The magnitude and length scales of the current are similar to those observed in the nearshore environment. However, this problem differs from the nearshore in that there are no breaking waves driving the flow and instead the tides are the forcing mechanism for the currents. The currents are also bounded by land on both sides of the flow through the inlet versus only on one side by the coast.

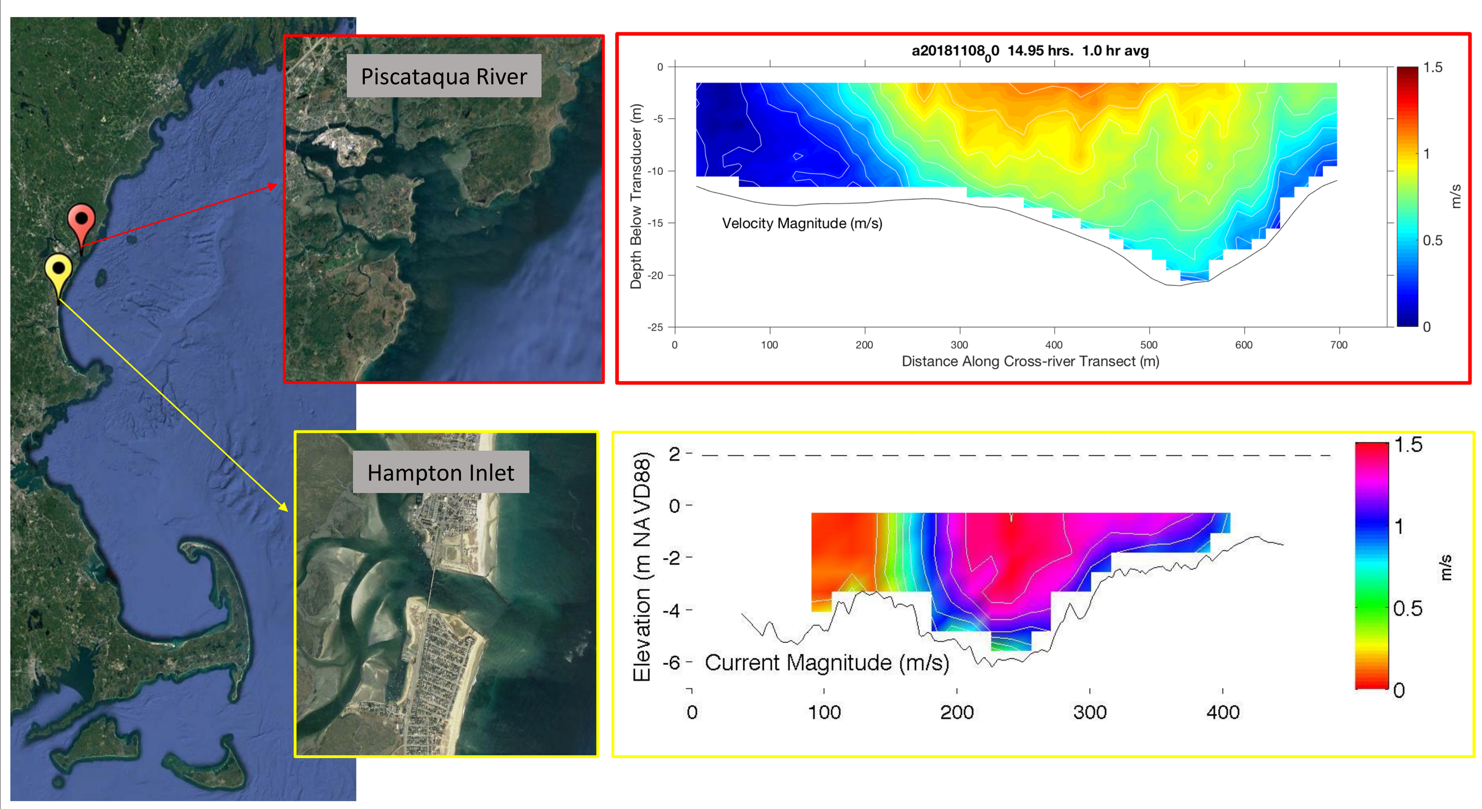


Figure 1. (Above) Tidal current observations (m/s) collected by Lippmann et al. in the Piscataqua River (red, top) and Hampton Inlet (yellow, bottom) along a transect across each inlet (x-axis) versus depth (y-axis).

The horizontal shear of the mean current can cause the flow to become unstable leading to a barotropic instability (i.e. shear or vorticity wave) that can lead to the development of eddies [1, 2, 3, 4]. In this work, we follow Bowen and Holman (1989) to analytically solve for linear shear instabilities in ebb and flood tidal currents in narrow estuarine channels. The wave-like solutions have phase speed c = σ/k with σ and k the radian frequency and wavenumber, respectively. When σ has an imaginary solution, an instability develops with exponentially growing amplitude [1, 3]. The criteria for unstable solutions are an extremum of the background potential vorticity (Figure 2) and the pressure (or η) must be continuous across the regions [1, 4]. An array of ADCP's will be deployed in the estuary to obtain estimates of σ-k spectra that will be compared with the linear analytical solutions for the fastest growing modes (Figure 3). Numerical models may be run to determine if the instabilities develop into non-linear eddies.

Methods

We start with the linearized, depth-integrated, shallow water, inviscid, horizontal momentum equations. The total flow is a combination of the mean flow plus a small perturbation, u' = (u, v + V) where u, v << V, and is represented by the stream function, ψ. Under the rigid-lid assumption with non-divergent flow, and neglecting the Coriolis force, the horizontal momentum equations can be represented as the conservation of potential vorticity equation (Eq. #1). The solution for the stream function is below (Eq. #2), which reduces to the Rayleigh Equation (Eq. #3) in each region (Figure 2).

$$(V - C_p)\left(\psi_{xx} - k^2\psi - \frac{\psi_x h_x}{h}\right) - h\psi\left(\frac{V_x}{h}\right)x = 0 \quad \text{(Eq. \#1)}$$

$$\psi = exp(\sigma_{im} t)\ \mathcal{Re}\{\psi(x)\ exp[i(ky - \sigma_{re} t)]\} \quad \text{(Eq. \#2)}$$

$$\psi_{xx} - k^2\psi = 0 \quad \text{(Eq. \#3)}$$

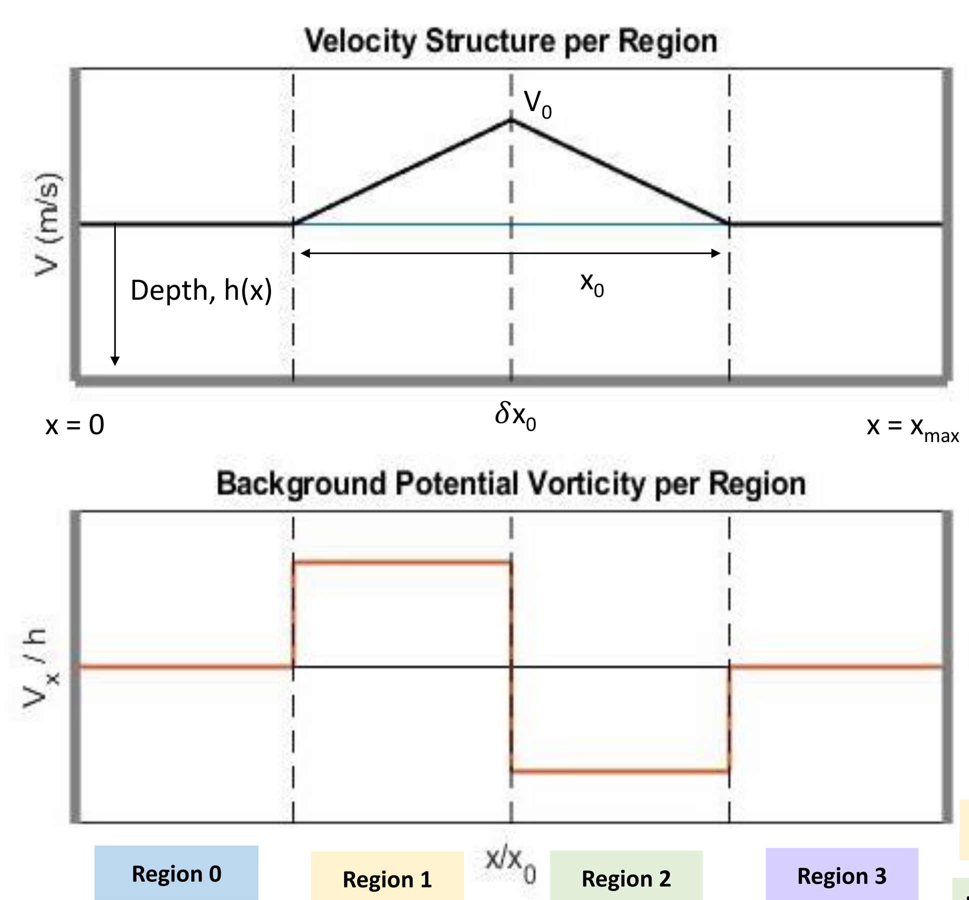


Figure 2. (Left) Following Bowen and Holman (1989), the along-channel tidal current structure (black line) over a flat bottom, h, (gray bar) and associated background vorticity, $\frac{V_x}{h}$, (orange line) are sketched per region across a channel, which is bounded on either end, x = 0 and x = x_{max}, (gray bars). The solutions for each region are shown below.

Reg. 0 : $\psi_0 = A_0 \sinh(kx)$

Reg. 1: $\psi_1 = A_1 \sinh(kx) + B_1 \cosh(kx)$

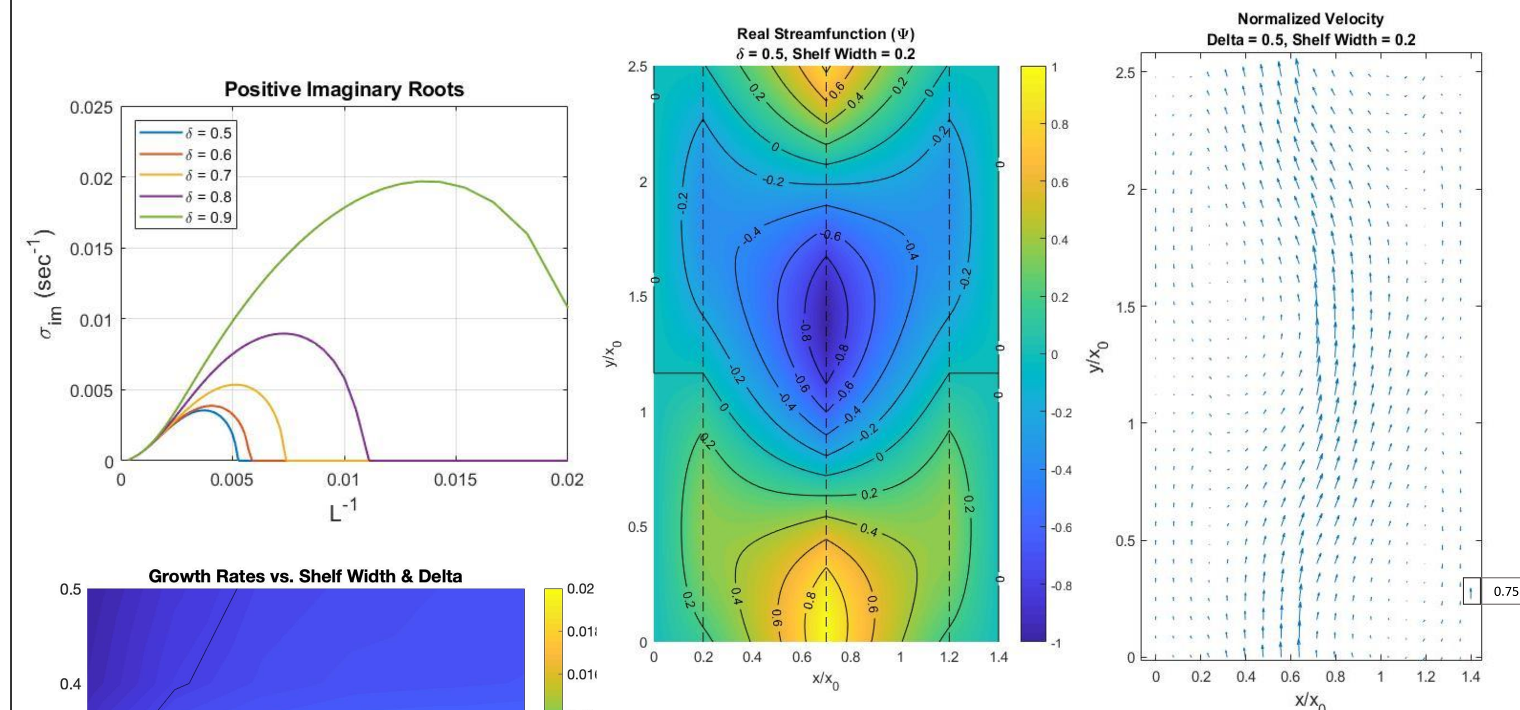
Reg. 2: $\psi_2 = A_2 \sinh(kx) + B_2 \cosh(kx)$

Reg. 3: $\psi_3 = A_3 (\sinh(kx) - \tanh(kx_{max})\cosh(kx))$

The maximum current (V_0) was set to 1 m/s and the current width (x_0) was set to 100 m. The location of the maximum current varied as a function of current width, δx_0, which changed the strength of shear on either side of the maximum. The width of regions 0 and 3 (i.e. the shelves) varied as a function of the current width (x_0). After matching conditions at the region boundaries, the dispersion relation was found to be cubic over both a constant depth (shown above) and for varying bathymetry, h(x), which includes shallow shelves, αh, in regions 0 and 3 surrounding a deeper channel, h, that runs through regions 1 and 2.

Figure 3. (Left) Acoustic Doppler Current Profiler (ADCP) mounted in a tripod. (Right) Lagged array of ADCPs that will be deployed in Hampton Inlet. Observations will be compared to the analytical solutions.

Results



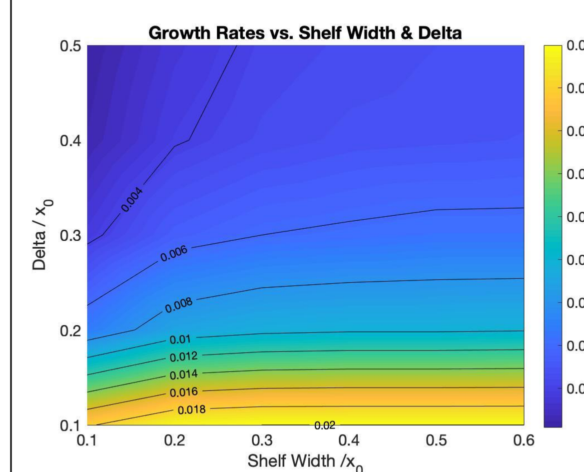


Figure 4. Positive imaginary roots to the linear instability problem (top left). The stream function and resulting velocity given δ = 0.5 and shelf width = $0.2x_0$. Velocity vectors are normalized to 0.75 m/s. (top middle and right). The growth rate as a function of strength of shear and shelf width (bottom left).

The analytical solutions to the linear instability problem over a constant depth are shown above. An increase in shear (δ) leads to faster growth rates of the unstable shear wave modes and a larger range of unstable wavelengths (Fig. 4, top left). The stream function shows how the shear wave progresses along the tidal inlet (Fig. 4, top middle). The derivative of the stream function results in the total velocity, which meanders along the inlet as the current becomes unstable (Fig. 4, top right). The fastest growing wave mode and wave lengths of the linear instabilities are both sensitive to the strength of shear of the current and the shelf width (Fig. 4, bottom). An increase in shear causes faster growth rates and smaller wavelengths. The growth rate and wavelengths become insensitive to changes in shelf width at a certain point.

Implications

It is expected that shear instabilities will cause tidal currents in narrow inlets and estuaries to meander along the channels varying in both time and space. This variability and any eddies or vortices produced by nonlinear instabilities may impact navigation and/or the passive transport of both organic and inorganic substances (e.g. nutrients, larvae, oil). Under the rigid lid assumption, in order for an instability to occur energy must be transferred from the background flow (V) to the perturbed flow (u',v') through the Reynolds stress $\rho\overline{(u'v')}$ [2]. Non-zero Reynolds stresses can be observed with ADCPs and determine the strength of the mixing of momentum in the estuary by instability mechanisms. Consequences include impacts on renewable energy initiatives as mean kinetic energy would be lost to turbulent kinetic energy through development of unstable eddies.

References

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